



Mass loss recipes over stellar evolution

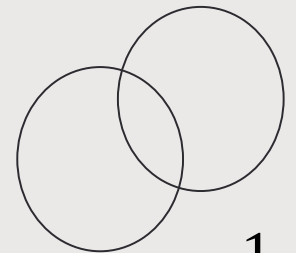
International Summer School
Stellar Winds and Outflows

Julieta P. Sanchez Arias

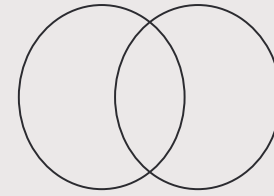


**Physics of Extreme
Massive Stars**

Marie-Curie-RISE project
funded by the European Union



Índice

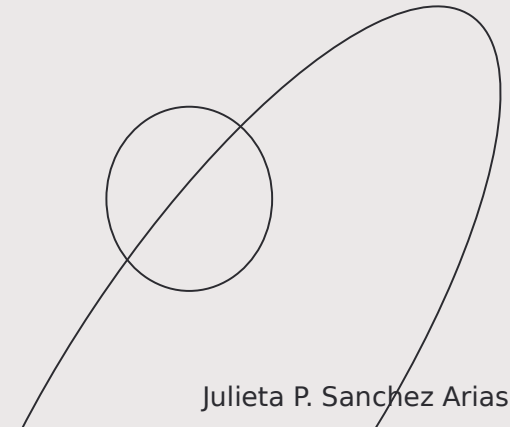


01 **Intoduction**

02 **Mass loss recipes**

03 **Summary**

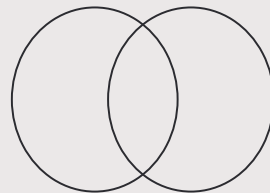
04 **Stellar oscillations**





01

Introduction



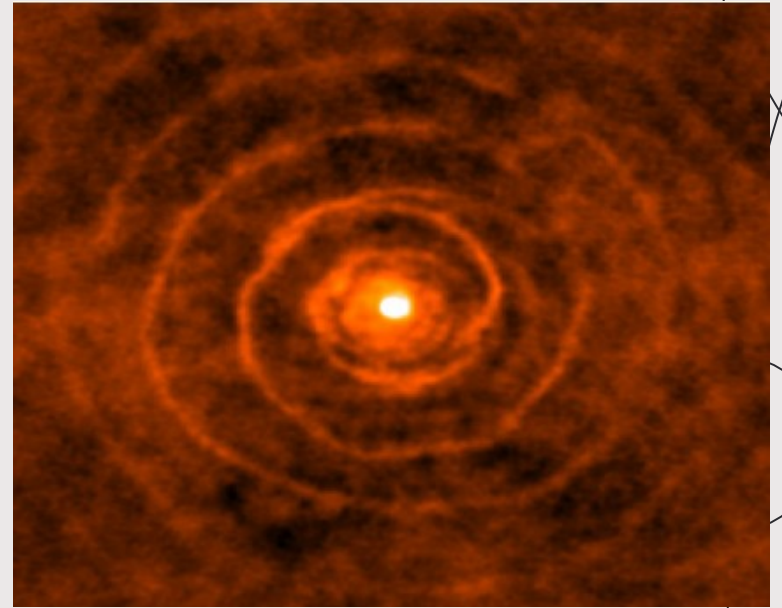
Mass loss process gain relevance at different evolutionary stages



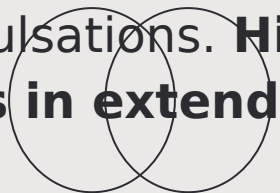
- Sun mass loss = 10^{-14} Msun/year (via solar winds)
- **Highest mass loss** are known for very massive stars ($M > 50$ Msun) and intermediate mass (~ 5 Msun) at very late evolutionary stages

Nuclear process which provide part of the radiation lost from stellar surface, imply a conversion from matter to energy and **leads to a reduction of the stellar mass, too.**

Stellar winds results from the **interaction** of the **photons** emitted from the photosphere with **atoms, molecules or dust grain** in the atmosphere → Complicated **radiation-hydrodynamic problem** that may depends in addition on chemical process.



Winds from **very cool stars** depend on the coupling of radiation to dust grains. Their formation process strongly depends on the **temperature and density** in their atmosphere and might be subject of regular variations due to stellar pulsations. **High mass loss rates are often related to pulsations in extended stellar envelopes.**



- **Full theoretical model** for any stellar wind **is not available**
- **Information** about stellar mass loss still results from **observations**

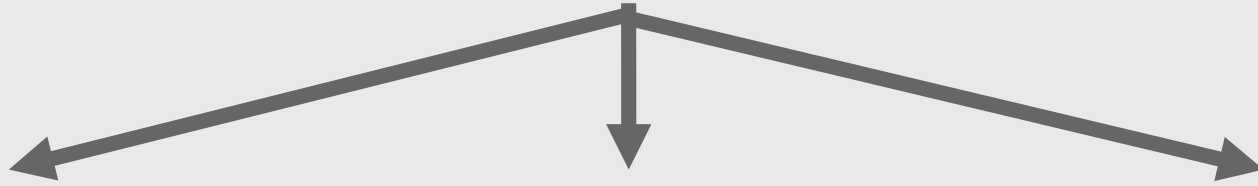


Empirical mass loss formulations are used in stellar evolution models.

They all have been obtained from observations of some **class of stars** and therefore **differ from each other**.

None of them is very accurate but it suffices to have the **correct order of magnitude** of mass loss and its **dependence on the global properties** of the star.

- Mass loss in stellar evolution models



Radiative
mass loss

Supra-Eddington
mass loss

Mechanical
mass loss

- Mechanical mass loss:

The mass per unit of time lost equatorially when the **surface velocity** of the star reaches the **critical velocity**.

The mechanical mass loss is determined by the **angular momentum** that needs to be lost to ensure that the surface velocity remains **subcritical**.

• Supra-Eddington mass loss

Eddington limit
($\Gamma = 1$): point at which a star's luminosity is so strong that the radiation force balances gravity

Eddington luminosity is the maximum luminosity a star can achieve when there is **balance** between the **force of radiation** acting outward and the **gravitational force** acting inward.

For some massive stars models ($>15M_{\text{sun}}$) in the RSG phase some external layers of the envelope might exceed the Eddington luminosity $L_{\text{Edd}} = 4\pi cGM/\kappa$, due to a peak in the **opacity** that lead to **lower L_{Edd}** \rightarrow to increase artificially the mass loss rate (in Geneva models: by a factor of 3 whenever $L > 5 * L_{\text{Edd}}$)

• Radiative mass loss:

Recipes

- Vink et al. (2001)
- de Jager et al. (1988)
- Reimers (1977)
- Sylvester et al. (1998) & van Loon et al. (1999)
- Nugis & Lamers (2000)

Correction for rotating models:

$$\dot{M}(\Omega) = F_{\Omega} \dot{M}(\Omega = 0) = F_{\Omega} \dot{M}_{\text{rad}}$$

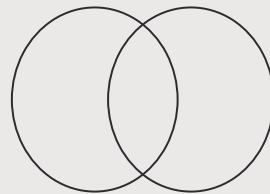
$$F_{\Omega} = \frac{(1 - \Gamma)^{\frac{1}{\alpha} - 1}}{\left[1 - \frac{\Omega^2}{2\pi G \rho_m} - \Gamma\right]^{\frac{1}{\alpha} - 1}}$$

$$\Gamma = L/L_{\text{Edd}}$$



02

mass loss recipes



Vink et al (2001)

- They studied the mass loss in **early O & B type of stars**
- In the relevant spectral range in which **early-type of stars** emit most of their radiation, **H & He** have only few lines & **metal lines** are responsible for the line driving.
- $\dot{M} = f(Z)$ for $1/100 < Z/Z_{\text{sun}} < 10$.
- They took into account momentum transfer of radiation to gas in a way that allows **photons interact with ions** more than once. (**multiple scattering**)

Vink et al (2001)

- Castor et al. 1975, predicted a power-law without the effect of "multiple scattering"
- But such pure power-law dependence of \dot{M} on Z over the entire parameters space is **questionable**, due to the presence of one or more "**bi-stability**" jumps.
- The ionization equilibrium depends on Temperature and Density- → the **position of this bi-stability jump might be shifted as function of Z** .
- From theory and observations: $v_{\infty}/v_{\text{esc}}$ jumps from 2.6 at the hot side of the jump to 1.3 to the cool side.

$$\dot{M} \propto Z^m$$

Vink et al (2001)

- Monte Carlo simulations to **follow the fate** of a large number of **photons** through the wind and calculates the **radiative acceleration of the wind material**.
- Non-LTE approximation for the model of extended atmospheres with ISA-WIND code.
- For each Z , they calculated \dot{M} for 12 temperatures between 12500K and 50000K
- $X = 1 - Y - Z.$ $Y = Y_p + \left(\frac{\Delta Y}{\Delta Z}\right) Z$ With $Y_p=0.24$ and $DY/DZ=3$

Vink et al (2001)

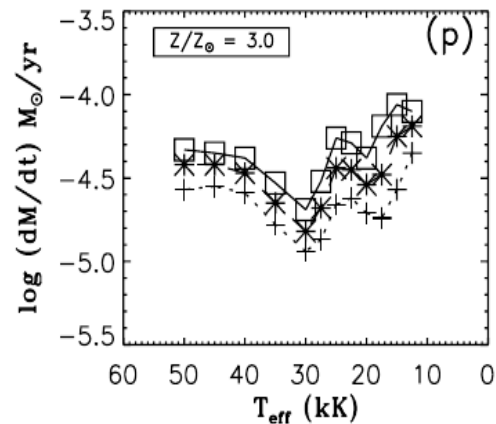
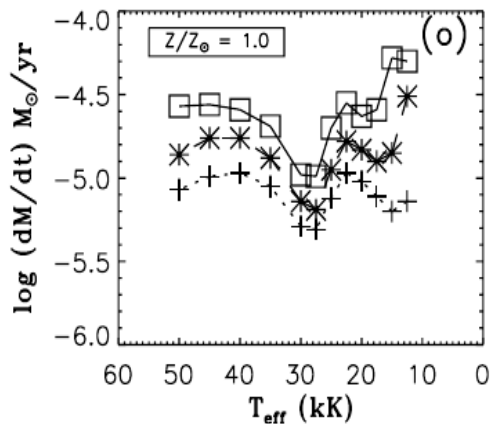
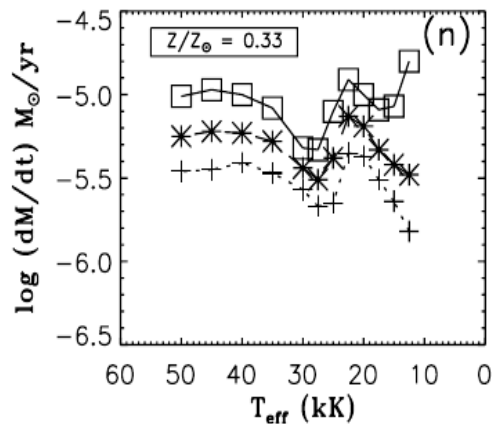
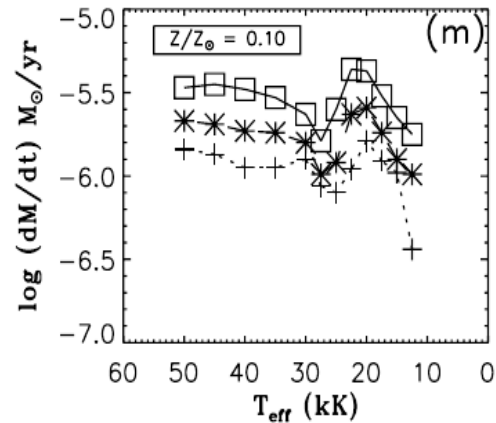
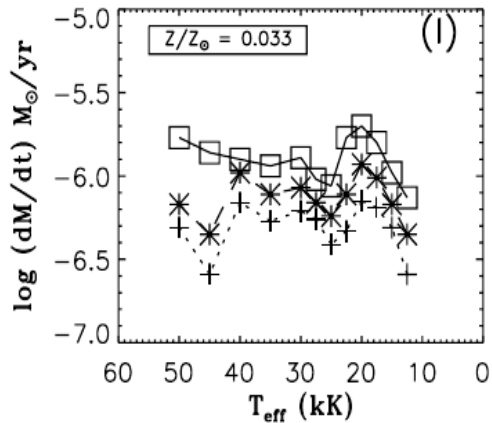
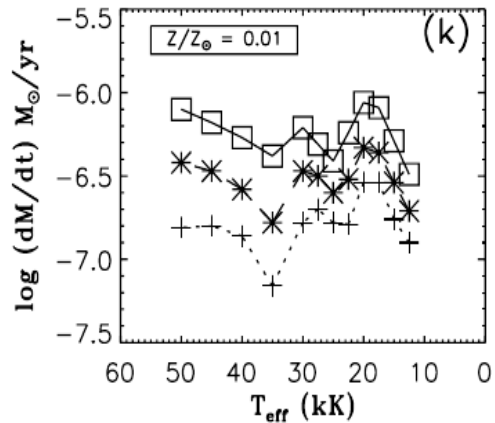
- 3 different values of the Eddington factor to explore the dependency for different L and M.

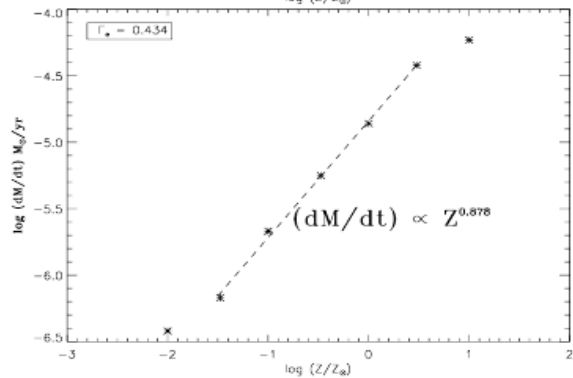
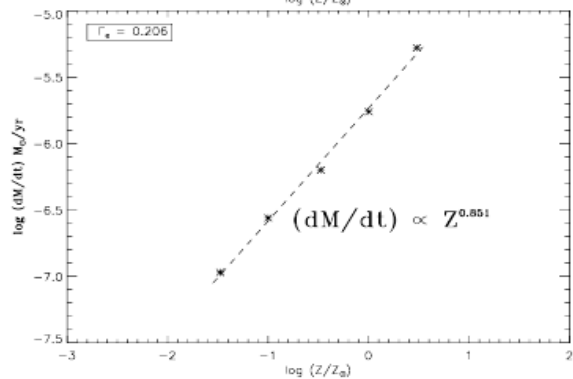
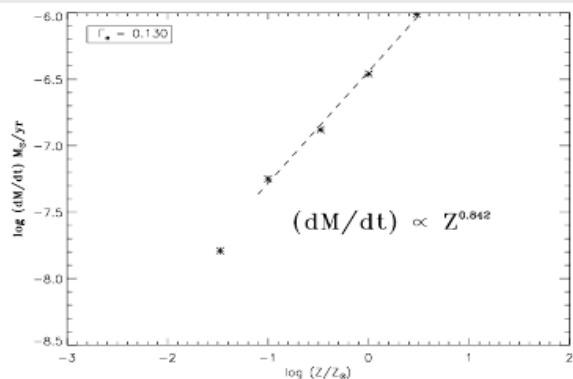
$$\Gamma_e = \frac{L\sigma_e}{4\pi cGM} = 7.66 \cdot 10^{-5} \sigma_e \left(\frac{L}{L_\odot}\right) \left(\frac{M}{M_\odot}\right)^{-1}$$

- electron scattering cross-section per unit of mass

- Different values for the ratio of the terminal velocity over the escape velocity
- And considered a beta-type velocity law for the accelerating part of the wind

$$v(r) = v_\infty \left(1 - \frac{R_*}{r}\right)^\beta$$





$$\begin{aligned} \log \dot{M} = & - 6.697 (\pm 0.061) \\ & + 2.194 (\pm 0.021) \log(L_*/10^5) \\ & - 1.313 (\pm 0.046) \log(M_*/30) \\ & - 1.226 (\pm 0.037) \log\left(\frac{v_\infty/v_{\text{esc}}}{2.0}\right) \\ & + 0.933 (\pm 0.064) \log(T_{\text{eff}}/40\,000) \\ & - 10.92 (\pm 0.90) \{\log(T_{\text{eff}}/40\,000)\}^2 \\ & + 0.85 (\pm 0.10) \log(Z/Z_\odot) \end{aligned}$$

for $27\,500 < T_{\text{eff}} \leq 50\,000$ K

$$\begin{aligned} \log \dot{M} = & - 6.688 (\pm 0.080) \\ & + 2.210 (\pm 0.031) \log(L_*/10^5) \\ & - 1.339 (\pm 0.068) \log(M_*/30) \\ & - 1.601 (\pm 0.055) \log\left(\frac{v_\infty/v_{\text{esc}}}{2.0}\right) \\ & + 1.07 (\pm 0.10) \log(T_{\text{eff}}/20\,000) \\ & + 0.85 (\pm 0.10) \log(Z/Z_\odot) \end{aligned}$$

for $12\,500 \leq T_{\text{eff}} \leq 22\,500$ K

de Jager et al (1988)

- They performed an **statistical analysis** of the mass loss in 271 star of **spectral types O through M**
- Until then the recipes depended on **M, g and R** which cannot be **directly observed**. Their recipe depends on **T_eff and L** → the knowledge of M, g, R may be essential for understanding the **mechanism** which produce the mass loss.
- The \dot{M} depends on the terminal velocity and **v_inf** can be different for stars **with the same T_eff, L, and M** (when the radiation is not the main agent for driving winds).

de Jager et al (1988)

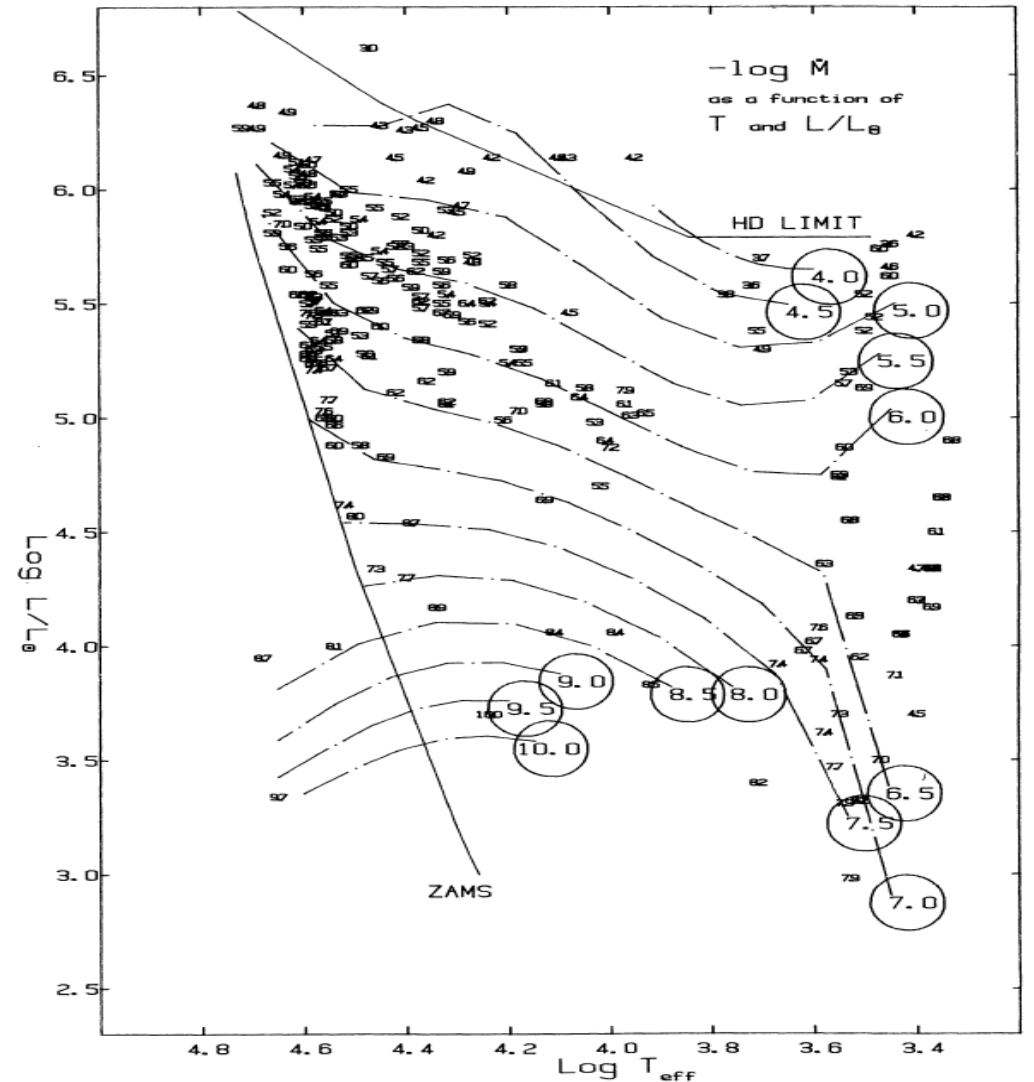
- They compile **6 different methods** to derive the mass loss of these stars in the literature (including from the UV spectra, from H_alpha, from the infrared)
- They calculate **an average** for the values of \dot{M} obtained with the same methods but different dataset.
- They adopted the **T_eff and L as the average from individual measurement** and the **statistical relation between spectral type and luminosity class+T_eff and L**

$$-\log(-\dot{M}) = \sum_{n=0}^N \sum_{\substack{i=0 \\ j=n-i}}^{i=n} a_{ij} T_i(\log T_{\text{eff}}) \times \\ \times T_j\left(\log\left(\frac{L}{L_{\odot}}\right)\right)$$

At first order:

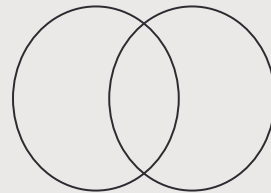


$$\log_{10}(-\dot{M}) = 1.769 \log_{10}(L/L_{\odot}) - 1.676 \log_{10}(T_{\text{eff}}/[K]) - 8.158$$



Nieuwenhuijzen & de Jager et al (1990)

- Improvement over de Jager et al (1988) → they used **the same data sample** with similar methods but includes the dependency of the mass loss on the **total stellar mass**.
- They also translate the **temperature dependency into a radius** dependence.
- They used different **evolutionary models** to obtain the masses → their algorithm depends **on the evolutionary tracks** and how they were created (i.e. different mixing theories)



Nieuwenhuijzen & de Jager et al (1990)

- Also, stars at **different evolutionary stages** can pass through the **same point in the HR with different masses** → they derived a ‘average expected mass’ taking into account:

$$t^{(d)} \stackrel{\text{def}}{=} \frac{\delta t}{\sqrt{[\delta \log_{10}(T_{\text{eff}}/[\text{K}])]^2 + [\delta \log_{10}(L/L_{\odot})]^2}}$$

- Time for a star to travel over the distance (δT , δL)

- Resulting :

$$\log_{10}(-\dot{M}) = -14.02 + 1.24 \log_{10}(L/L_{\odot}) + 0.16 \log_{10}(M/M_{\odot}) + 0.81 \log_{10}(R/R_{\odot}) .$$

Van Loon et al. (2005)

- Empirically determined on the basis of observations of **oxygen rich AGB and RSG in the Large Magallanic Cloud**.
- AGB and RSG have very **extended and cool envelopes** where dust grains might form through sublimation → they based their analysis in **dust -driven wind model**.
- **Photons from radiation** transfer momentum to these grains, **pushing them away**. Gas grain drag the gas with them through collisional coupling.

Van Loon et al. (2005)

- They fit the observed IR spectra to synthetic spectra obtained with simple model of gas/dust mixture using T_{eff} and L as variable.

$$\log_{10}(-\dot{M}) = -5.65(15) + 1.05(14) \log_{10}(L/10^4 L_{\odot}) + \\ -6.3(1.2) \log_{10}(T_{\text{eff}}/3500 \text{ K}) ,$$

- Limitations: **high uncertainty of the dust grain properties** (mass fraction, opacity, when they form, etc)

Nugis & Lamers (2000)

- Only for WR stars. The amount of **He** their atmospheres **affects their temperature** and therefore the ionization fraction and the level population of all other atoms and ions.
- The wind mass loss rate of these stars **depends strongly on their chemical composition.**
- They considered a relevant sample of observed galactic WR stars.
 - 1 set: with **mass and distance known** (also L)
(binaries, open cluster)
 - 2 set: with **unknown** intrinsic **luminosity**

Nugis & Lamers (2000)

- They derived an **empirical bolometric** correction for set 1.
- They used a **theoretical Mass-Luminosity relation** to infer the luminosity of set 2 and then corrected it by the bolometric correction.

$$\log_{10}(-\dot{M}) = -11.0 + 1.29(14) \log_{10}(L/L_{\odot}) + 1.73(42) \log_{10}(Y) + 0.47(09) \log_{10}(Z) ,$$

- **WR winds are strong and optically thick** → the radius is a **function of the wavelength** → It cannot be expressed in terms of **R** or **T_{eff}** (no Stefan-Boltzmann law).

Kudritzki et al (1989)

- Based on an **analytic solution** for stationary, isothermal, spherically-symmetric and without viscosity **gas flow** with no magnetic fields and no rotation.
- To find an analytic solution they assumed a solve “ $\beta-1$ law” for the velocity field:

$$v(r) = v_{\infty} \left(1 - \frac{R}{r}\right)^{\beta}$$

- Common assumption close to numerical solutions ($\beta=1$)
- They **do not treat dynamically** the system, but rather assume a velocity structure.

Kudritzki et al (1989)

- Free parameters

$$\dot{M} \equiv \dot{M}(\alpha, \delta, k, M, L, v_{th}) = \tilde{D}(\alpha, \delta, v) \left(\frac{\sigma_{Th} k L}{4\pi c} \right)^{1/(\alpha-\delta)} \times \\ \times \left(\frac{4\pi\alpha}{\sigma_{Th} v_{th}} \right)^{\alpha/(\alpha-\delta)} \left(\frac{1-\alpha}{GM(1-\Gamma)} \right)^{(1-\alpha)/(\alpha-\delta)},$$

- Thermal velocity of protons

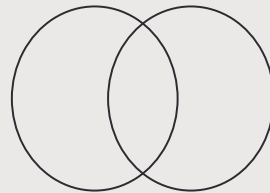
- Eddington ratio = L/L_{Edd}

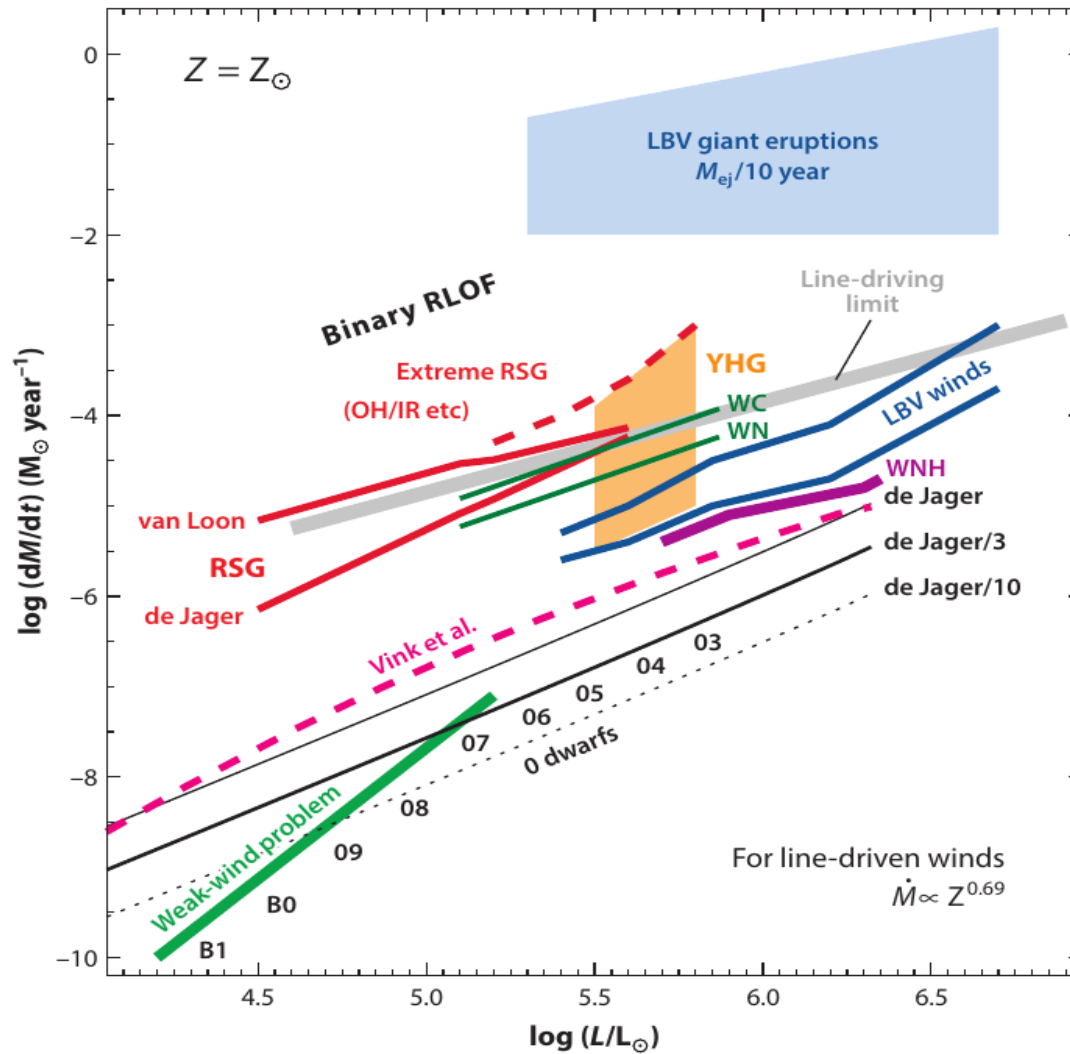
- **Caveats:** the ‘free parameters’ are not constant but rather depends on the **optical depth**. Practically, **they are calibrated on ξ Pupis**.



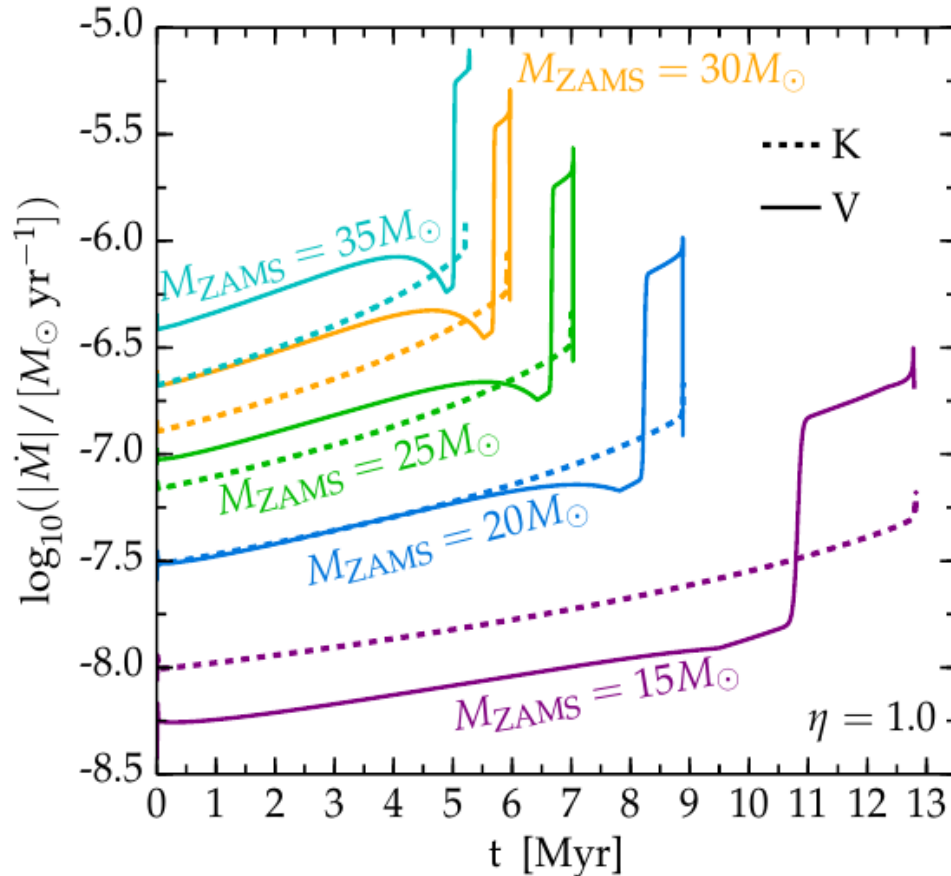
03

Summary





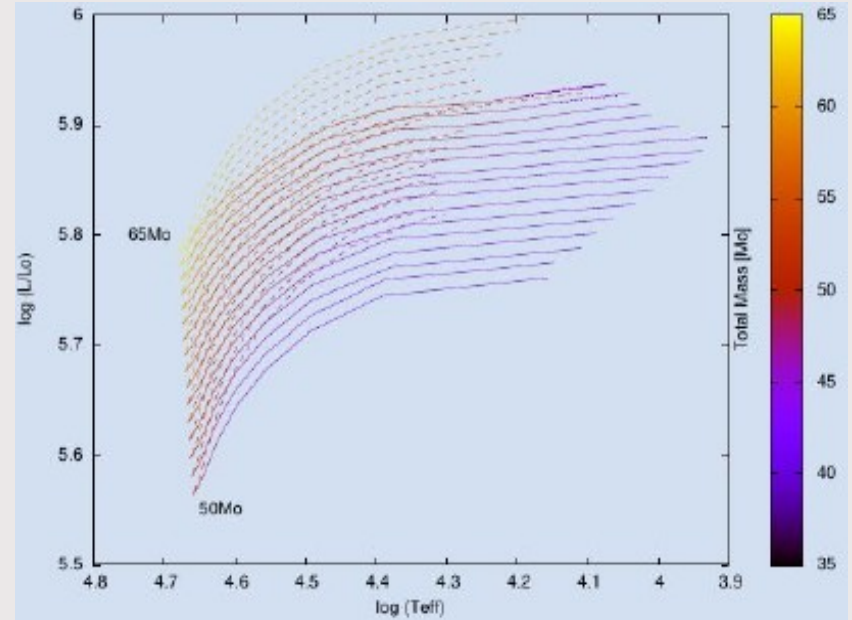
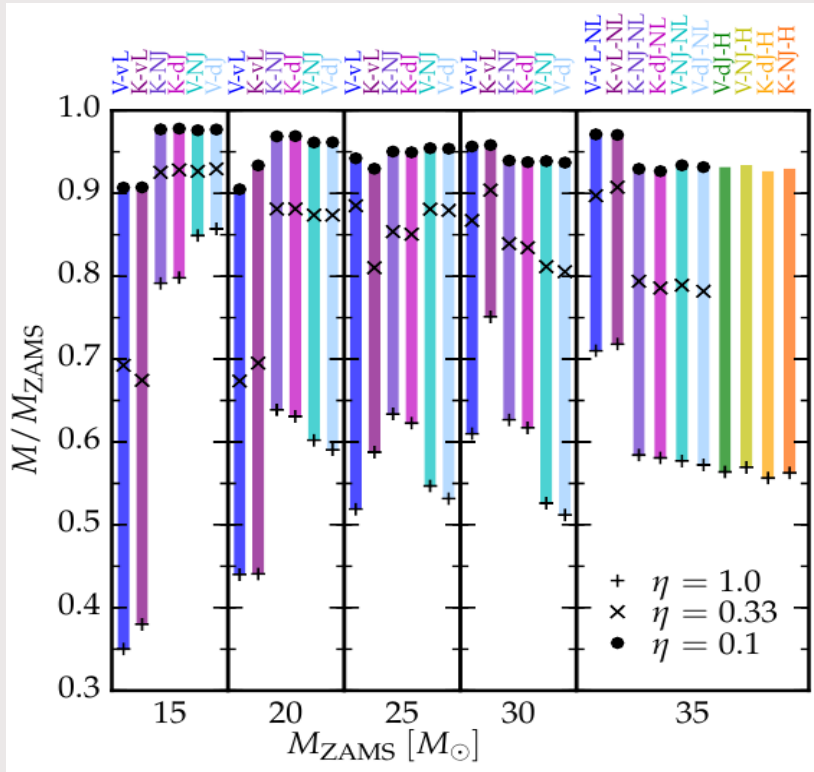
- Comparison between Vink and Kudritzki



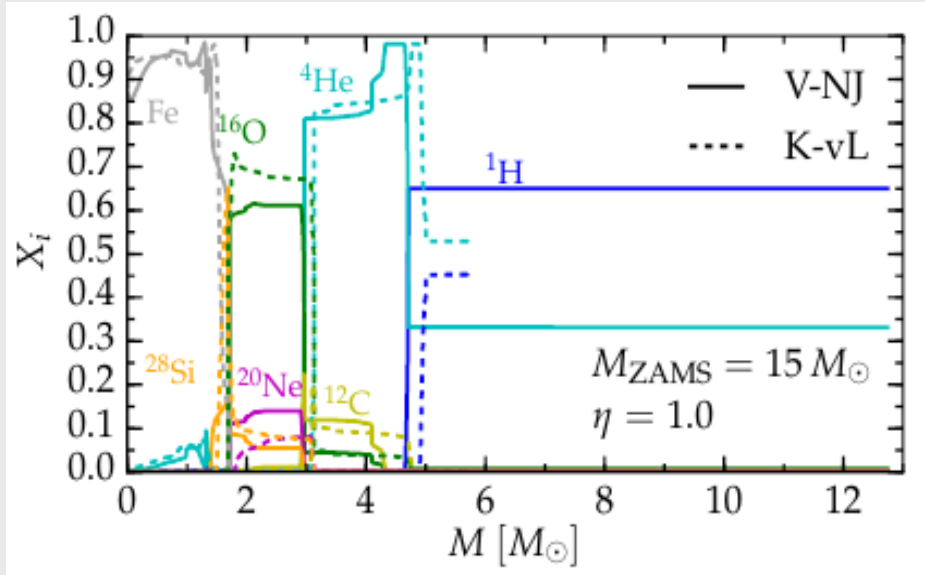
- The rapid rise in the solid curves is due to the inclusion of the bistability jump.

- Parameter to modify the wind efficiency, to take into account the effect of clumping, for example.

- Efficiency parameter



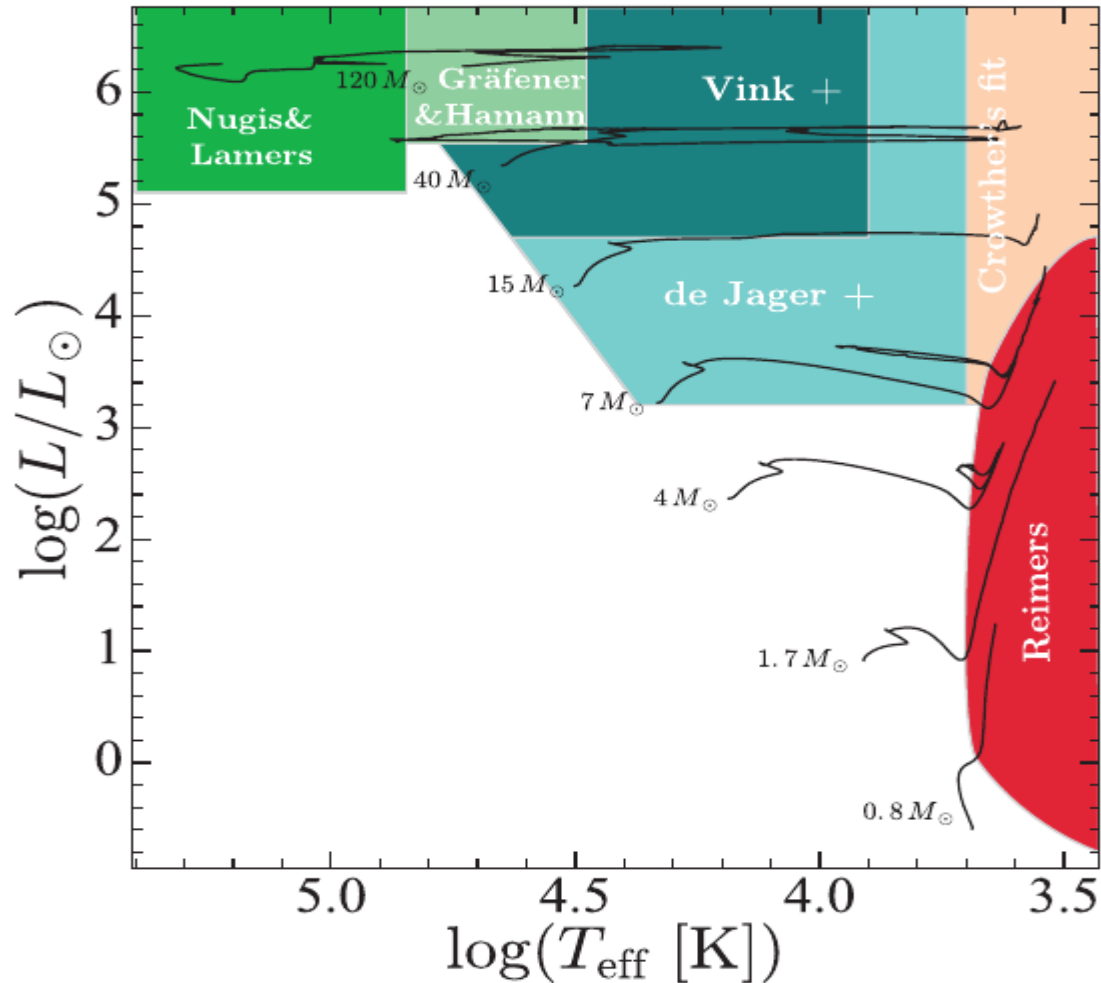
- Results by different recipes applied over the evolution



- Chemical profile for a pre-SN at the onset of the core collapse
- An initial 15Msun star can reach the onset of the core collapse as a He rich 6 Msun or as a 12 Msun H rich star



- It directly affect the enrichment of the interstellar medium.



• On main sequence:

- Stars with mass below 7 Msun → constant mass
- Above 7 Msun → Radiative mass-loss rate is adopted from Vink et al. (2001)

• For red (super) giants:

- Stars up to 12 Msun → Reimers(1975,1977)
- for stars of 15 Msun and above → de Jager et al. (1988)
constant mass (**for $\log(T_{\text{eff}}) > 3.7$**) & linear fit to the data from Sylvester et al (1998) & van Loon et al. (1999)
- (**for $\log(T_{\text{eff}}) < 3.7$**)

• For Wolf-Rayet:

- Evolved stars with $M > 20-30 M_{\text{sun}}$, $35000 < T_{\text{eff}} < 50000$ → Nugis & Lamers (2000), or Grafener & Hamann (2008) in its small domain.
- When \dot{M} (Grafener & Hamann (2008)) $<$ \dot{M} (Vink et al. (2001)) → Vink recipe is used.
- Nugis & Lamers (2000) and Grafener & Hamann (2008) mass loss rates account for some clumping effects (Muijres et al (2011) making them 2 or 3 times lower the normal rates.

• Caveats

- There remain **substantial issues in understanding the physics of wind driving, magnetic field and angular momentum loss** and is challenging to understand how these corresponds to observational diagnosis.
- Usually, **mass loss rates** needs to be **reduced** by a factor of 2(3) compared with rates derived **observationally** (like those of de Jager & Nieuwenhuijzen) → is like replace mass loss rates at $Z=Z_{\text{sun}}$ by currently used mass loss rates for $Z=0.37(0.2) Z_{\text{sun}}$
- There is a high level of uncertainties in the most important phases of mass loss for massive stars → • **This isn't good!**

• What can we do?

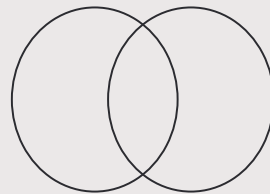
- DON'T PANIC!
- We have a firm understanding of stellar winds of **hot O-type** of stars relevant for most of their lives **on the H-burning MS phase**
- To place more emphasis on **matching individual observed stars** with well-constrained physical parameter, instead of matching statistics of massive-stars populations which are **contaminated by binaries.**
- To use stellar evolution codes as **“toy models”** to investigate the final outcome for a wide range of mass loss including episodic mass loss





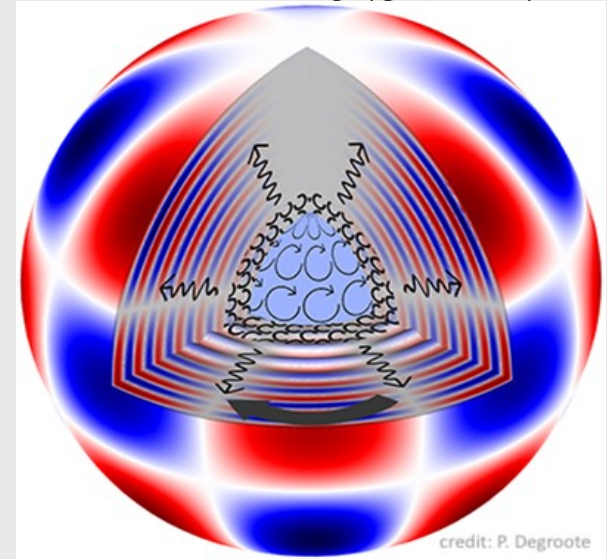
04

Oscillations



Stellar oscillations

- Pulsations can facilitate the mass loss in massive stars.
- In classical pulsators, pulsations are usually excited by the κ -mechanism.
- Modes known to facilitate mass loss are driven mainly by **instabilities of non-thermal origin** and can be interpreted in term of **mechanical quantities** only.



Stellar oscillations

- Adiabatic equations

$$\frac{\delta \rho}{\rho} = -\vec{\nabla} \cdot \delta \vec{r},$$

$$T \frac{\partial \delta s}{\partial t} = \delta \left(\epsilon - \frac{dL}{dm} \right)$$

$$\frac{\partial^2 \delta \vec{r}}{\partial t^2} = -\vec{\nabla} \psi' - \frac{\vec{\nabla} P'}{\rho} + \frac{\rho'}{\rho} \vec{\nabla} \psi,$$

$$\frac{\delta P}{P} = \Gamma_1 \frac{\delta \rho}{\rho} + \frac{\rho}{P} (\Gamma_3 - 1) T \delta s$$

$$\nabla^2 \psi' = 4\pi G \rho'$$

$$x \frac{dy_1}{dx} = \left(\frac{V}{\Gamma_1} - 1 - \ell \right) y_1 + \left(\frac{\ell(\ell+1)}{c_1 \omega^2} - \frac{V}{\Gamma_1} \right) y_2 + \frac{\ell(\ell+1)}{c_1 \omega^2} y_3 + v_T y_5,$$

$$x \frac{dy_2}{dx} = (c_1 \omega^2 - A^*) y_1 + (A^* + 3 - U - \ell) y_2 - y_4 + v_T y_5,$$

$$x \frac{dy_3}{dx} = (3 - U - \ell) y_3 + y_4,$$

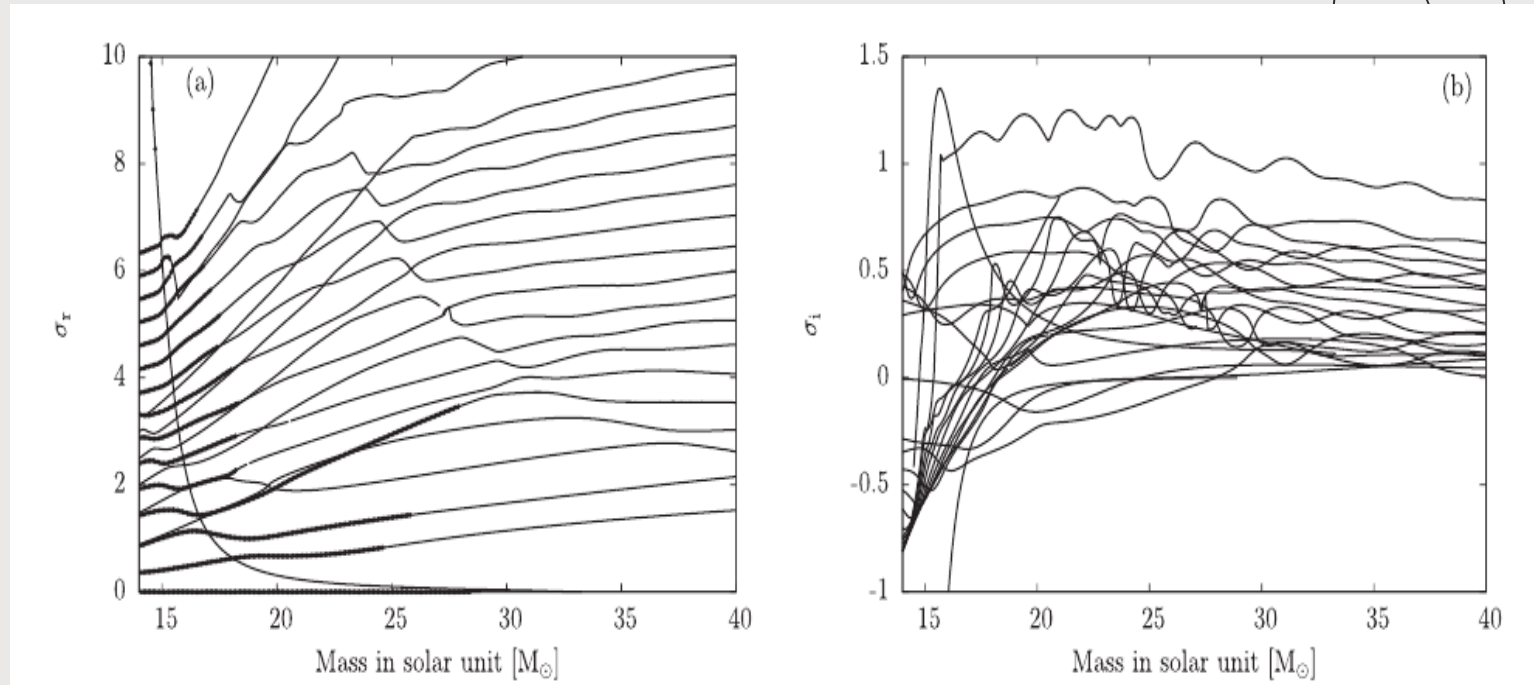
$$x \frac{dy_4}{dx} = U A^* y_1 + U \frac{V}{\Gamma_1} y_2 + \ell(\ell+1) y_3 + (2 - U - \ell) y_4 - v_T U y_5,$$

$$x \frac{dy_5}{dx} = V \left[\nabla_{\text{ad}} (U - c_1 \omega^2) - 4(\nabla_{\text{ad}} - \nabla) + c_{\text{dif}} \right] y_1 + V \left[\frac{\ell(\ell+1)}{c_1 \omega^2} (\nabla_{\text{ad}} - \nabla) - c_{\text{dif}} \right] y_2 \\ + V \left[\frac{\ell(\ell+1)}{c_1 \omega^2} (\nabla_{\text{ad}} - \nabla) \right] y_3 + V \nabla_{\text{ad}} y_4 + [V \nabla (4 - \kappa_S) + 2 - \ell] y_5 - \frac{V \nabla}{c_{\text{rad}}} y_6,$$

$$x \frac{dy_6}{dx} = \left[\ell(\ell+1) c_{\text{rad}} \left(\frac{\nabla_{\text{ad}}}{\nabla} - 1 \right) - V c_{\epsilon, \text{ad}} \right] y_1 + \left[V c_{\epsilon, \text{ad}} - \ell(\ell+1) c_{\text{rad}} \left(\frac{\nabla_{\text{ad}}}{\nabla} - \frac{3 + \partial c_{\text{rad}}}{c_1 \omega^2} \right) \right] y_2 \\ + \left[\ell(\ell+1) c_{\text{rad}} \frac{3 + \partial c_{\text{rad}}}{c_1 \omega^2} \right] y_3 + \left[c_{\epsilon, S} - \frac{\ell(\ell+1) c_{\text{rad}}}{\nabla V} + i \omega c_{\text{thm}} \right] y_5 - (\ell+1) y_6.$$

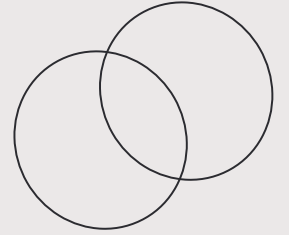
- Non-adiabatic equations

Stellar oscillations



Stellar oscillations

- Caveats:
 - It is not possible to quantify the mass lost by stellar oscillations
 - There is no open source code capable of **following these modes through a non-linear stability analysis.**





Thanks!

Bibliography

- https://www.aanda.org/articles/aa/full_html/2012/01/aa17751-11/aa17751-11.html
- <https://www.aanda.org/articles/aa/pdf/2014/04/aa22573-13.pdf>
- <https://ui.adsabs.harvard.edu/abs/2000A%26A...360..227N/abstract>
- <https://www.aanda.org/articles/aa/abs/2008/18/aa6176-06/aa6176-06.html>
- <https://ui.adsabs.harvard.edu/abs/1999MNRAS.303..116G/abstract>
- <https://articles.adsabs.harvard.edu/pdf/1977A%26A....57..395R>
- <https://arxiv.org/pdf/1703.09705.pdf>
- <https://arxiv.org/abs/2109.03239>

