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Physics of Extreme Massive Stars

Marie-Curie-RISE project funded by the European Union

Radiation Driven Winds

The radiation force

Lydia Cidale

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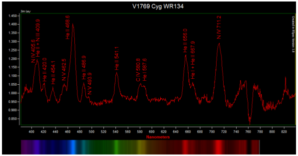
Emission Line Spectra

Although most stars display absorption lines or bands in their spectra, there are several **types of peculiar objects** that show evidence of **high-velocity outflows**.

Wolf-Rayet stars

- Broad emission lines and P Cygni profiles
- Extended shells
- Large scale velocity gradients and high terminal velocities (500-3000 km s^{-1})
- Deviations from local thermodynamic equilibrium



The **LBVs** $\rightarrow$ P Cygni profiles

The **Be stars** $\rightarrow$ emission lines and high rotation rates

Hot supergiants, symbiotic and nova-like stars, flare-type stars.

Emission lines cannot be modelled with a classical theory of stellar atmospheres (hydrostatic equilibrium and a plane-parallel approximation)

The velocity field has a strong impact on

- the line profiles
- the excitation state of the gas because the opacity of the medium becomes more transparent
- the radiation field

A moving medium increases the **probability that photons from deep layers escape** and, at the same time, the **radiative acceleration modifies the velocity regime (feedback).**

The great advance in the subject initiated with the escape-probability method to solve the radiative transfer problem and to compute the hydrodynamics for line-driven winds.

Hydrodynamics Equations for Spherical Winds

Mass conservation

$$\frac{\partial \rho}{\partial t} + \bar{\nabla} \cdot (\rho \bar{v}) = 0$$

Momentum conservation

$$\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \bar{\nabla})\bar{v} = -\frac{1}{\rho}\bar{\nabla} P + \bar{g} + \frac{1}{\rho}\bar{\mathbf{f}}$$

For a steady, spherically symmetrical wind, we have,

$$\boxed{\dot{M} = 4\pi r^2 \rho v}$$

$$\boxed{v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{GM_*}{r^2} + g_{rad}}$$

Two particular cases are $g_{rad} \propto \frac{1}{r^2}$ $\qquad g_{rad} \propto \frac{\partial v}{\partial r}$

Quantitative calculation of the radiation force

The **radiation force** per unit of volume and time exerted on a point particle is equal to the momentum removed from the incident radiation integrated over all the directions and frequencies,

$$\mathbb{F}(r,\bar{n}) = \frac{4\pi}{c}\int_0^\infty \kappa_\nu(\bar{r})\,\rho(\bar{r})\,F_\nu(\bar{r})\,d\nu, \quad \text{where} \quad F_\nu(\bar{r}) = \frac{1}{2}\int_{-1}^{+1} I_\nu(\bar{r},\mu)\,\mu\,d\mu$$

This expression depends on the medium's opacity at the distance $\bar{r}$, the photospheric intensity and emission and absorption processes between the photosphere and $\bar{r}$.
$\kappa_\nu(\bar{r})$ (in units of cm^2 g^{-1}) consists of three main contributions:

$$\kappa_\nu(\bar{r}) = \kappa^S(\bar{r}) + \kappa_\nu^C(\bar{r}) + \kappa_\nu^L(\bar{r})$$

- $\kappa^S(\bar{r})$: scattering coefficient (Compton/Thomson)
- $\kappa_\nu^C(\bar{r})$: continuum absorption (bound free and free-free) often neglected,
- $\kappa_\nu^L(\bar{r})$: line absorption coefficient (thousands of line transitions)

The Compton effect is the interaction between photons and free electrons. If ($\frac{1}{2}mv^2 \ll mc^2$), the frequency shift will be tiny. Small amounts of energy exchanged (repeated many times) can build up and produce substantial effects.

To properly calculate the **radiation force**,

$$\mathbb{F}_{\bar{r}} = \frac{4\pi}{c} \int_0^\infty \kappa_\nu(\bar{r})\, \rho(\bar{r})\, F_\nu(\bar{r})\, d\nu$$

we have to

- solve the TR in a moving medium → to compute **line opacities** (e.g., ∼ 500 000 line transitions and **line fluxes**
- compute the total radiation force
- solve the hydrodynamical equations → including the **radiation force**

Radiation force is not observable!

- compute a **wind model** (hydrodynamics + RT)
- compare the **theory with observations**

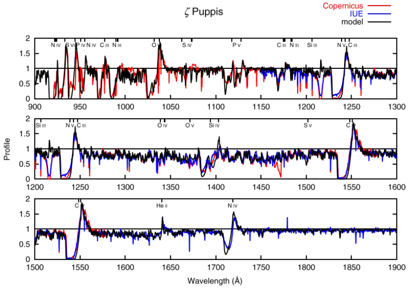


Figure 1: Calculated and observed UV spectrum for *zeta* Puppis (Pauldrach et al 2003)

Radiative Transfer in a moving medium

A straightforward approach is to formulate the **transfer equation** in the **inertial or observer's frame**.

A plasma volume moves with $\bar{v}(\bar{r})$ relative to an **external observer at rest**.

A photon (ν), travelling in direction $\bar{n}$, as measured in the observer's frame, has an atom's frame frequency ν',

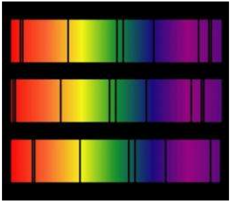
$$\nu' = \nu - \nu_0 \left(\bar{n} \cdot \bar{v}/c\right)$$

Disadvantage

- opacity and emissivity, as seen by **external observer** becomes **angle–dependent**

  $\kappa_\nu(\bar{r}, \bar{n})$ and $\eta_\nu(\bar{r}, \bar{n})$

Figure 2: **Doppler shift** between ν', measured in the observer's frame, and the local atom frame (ν_0) at rest.



$\rightarrow \nu_0$ at rest

$\rightarrow \nu' < \nu_0$ (redshifted)

$\rightarrow \nu' > \nu_0$ (blueshifted)

Radiative acceleration due to electron scattering

$$\bar{g}^{S}_{rad}(\bar{r}) = \frac{4\pi}{c}\int_0^\infty \kappa^{S}(\bar{r})\,\rho(\bar{r})\,F_\nu(\bar{r})\,d\nu$$

$$\kappa^{\mathrm{S}}\rho \;=\; n_e\,\sigma_{\mathrm{e}}$$

$\sigma_{\mathrm{e}} = 6.65\,10^{-25}$ is the Thomson cross-section

$\kappa^{\mathrm{T}} = 0.34\ \mathrm{cm}^2\ \mathrm{g}^{-1}$ (canonical value) for fully ionised plasma at solar abundance.

The contribution of the Thomson scattering to the radiative acceleration,

$$g^{\mathrm{T}} = \frac{4\pi}{c}\,n_e\,\sigma_{\mathrm{e}}\int_0^\infty F_\nu(\bar{r})\,d\nu = n_e\,\frac{\sigma_{\mathrm{e}}\,L}{4\pi c\,r^2}$$

L: the star's luminosity diluted by distance.

The radiative acceleration, as gravitational acceleration, are $\propto r^{-2}$

Radiative acceleration due to spectral lines

$$\bar{g}^{L}_{rad}(\bar{r}) = \sum_{lines}\frac{4\pi}{c}\int_0^\infty \kappa^{L}_\nu(\bar{r})\,\rho(\bar{r})\,F_\nu(\bar{r})\,d\nu$$

κ^L_ν is the line opacity coefficient between levels l (lower) and u (upper) with energy $\mathrm{h}\nu_0$

$$\kappa^L_\nu\,\rho \;=\; \frac{\pi e^2}{m_e c}\,f_l\,n_l\left(1 - \frac{n_u\,g_l}{n_l\,g_u}\right)\,\phi(\nu-\nu_0)$$

$$\kappa^L_\nu\,\rho = \chi^\ell_0\,\phi(\Delta\nu)$$

- n_l and n_u are the number density of ions in levels l and u (cm^{-3})
- g_l and g_u are the statistical weights
- f_l is the oscillator strength of the line
- $\phi(\nu-\nu_0)$ is the normalised line profile function

Different phenomena affect the line width.

Pressure effects

- Lorentzian profile

$$\phi(\Delta\nu)\, d\Delta\nu = \frac{(\gamma/4\pi)^2}{(\nu-\nu_0)^2+(\gamma/(4\pi)^2}\, d\Delta\nu$$

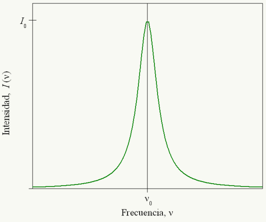


Figure 3: γ is the full width at half maximum (FWHM)

Temperature effects

- Gaussian profile

$$\phi(\Delta\nu)\, d\Delta\nu = \frac{1}{\sqrt{\pi}}\frac{1}{\delta\nu_D} e^{-\left(\frac{\nu-\nu_0}{\delta\nu_D}\right)^2}\, d\Delta\nu$$

$$\delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2KT}{m_e}} \quad \text{and} \quad \int_{-\infty}^{\infty}\phi(\Delta\nu)\, d\Delta\nu = 1$$

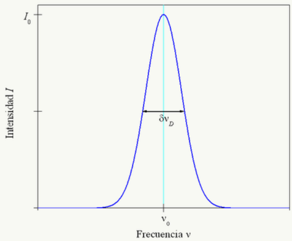


Figure 4: The width is given by the kinetic energy of particles (Maxwell-Boltzmann velocity distribution). At $\Delta\nu = 1.5\, \delta\nu_D$, the absorption coefficient is 10% the value in the line center.

Radiative acceleration due to spectral lines

$$g_{rad}^{L}(\bar{r}) = \sum_{\ell} \frac{4\pi}{c} \int_0^{\infty} \chi_0^{\ell}\, \phi(\Delta\nu)\, F_{\nu}(\bar{r})\, d\nu$$

The summation is over thousands of individual line transitions (ℓ) assuming non-overlapping lines for which the wind is optically thick.

$$\chi_0^{\ell} = \frac{\pi e^2}{m_e c} f_l\, n_l \left(1 - \frac{n_u\, g_l}{n_l\, g_u}\right)$$

- LTE $\frac{n_u}{n_l} \rightarrow$ Boltzmann distribution
- NLTE $\rightarrow \frac{dn_l}{dt} = 0$ (rate equations)

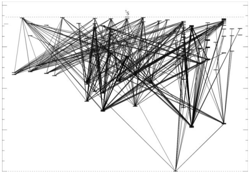


Figure 5: Energy level diagram of the most important levels of Mn I.

Partial Grothian Diagrams

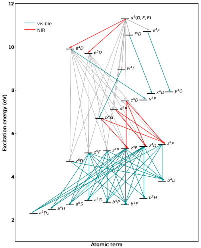
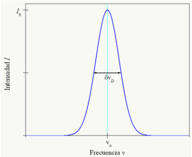


Figure 6: Energy level diagram of the most important levels of Fe II

The Line Interaction Region

A photon ν_p emitted by the star in direction θ' (i.e., along the path z) can be absorbed by the line transition if it encounters ions with a velocity v_z such that the Doppler shift brings ν_p within the line width in the atom's frame,



$$\nu_0 - 1.5\,\delta\nu_D \leq \nu_p\,(1 - v_z/c) \leq \nu_0 + 1.5\,\delta\nu_D$$

As $v_z \in (0, v_\infty)$, photons ν_p can interact with a line, along its path, in the interval

$\nu_0 - 1.5\,\delta\nu_D \leq \nu_p$ and

$$\nu_p \leq \frac{\nu_0 + 1.5\,\delta\nu_D}{(1 - v_\infty/c)} \simeq (\nu_0 + 1.5\,\delta\nu_D)\,(1 + v_\infty/c)$$

$$(\nu_0 - 1.5\,\delta\nu_D) \leq \nu_p \leq (\nu_0 + 1.5\,\delta\nu_D)\,(1 + v_\infty/c)$$

Outside this range, photons will not be absorbed!.

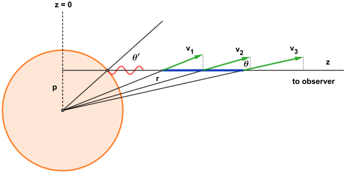
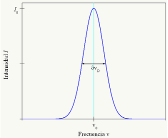


Figure 7: Line interaction region (solid blue line) is the region where the wind absorption occurs

The large extension for the interaction region complicates the calculation of the radiative force since it requires knowledge of the radiation field and the absorption coefficient.

Position r and velocity v_z where the absorption occurs!

$\nu_0 + \Delta\nu = \nu_p \left(1 - \frac{v_z}{c}\right)$



In a (z, p) coordinate system,

$$v_z(r) = v(r)\, cos\theta = \frac{z}{r}\, v(r)$$

$$\Delta\nu = \nu_p \left(1 - \frac{z}{r}\frac{v(r)}{c}\right) - \nu_0$$

Then, the size of the interaction region depends on the line width and velocity gradient

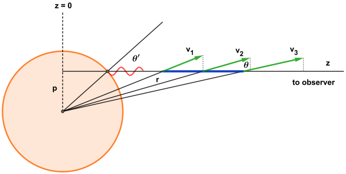
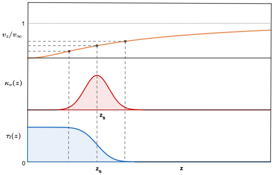


Figure 8: Line interaction region (solid blue line)

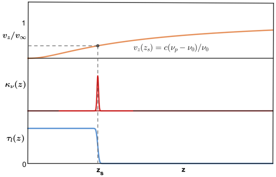
The Sobolev Approximation

The size of the interaction region:

$$\Delta\nu = \nu_p \left(1 - \frac{z}{r}\frac{v(r)}{c}\right) - \nu_0$$



if $\delta\nu_D \to 0$



$$\tau_{\nu_p}(z_1) = \int_{z_1}^{\infty} \kappa_{\nu_p}(z)\,\rho(z)\,dz$$

If the interaction region is very narrow, it is possible to simplify the radiative transfer in stellar winds. V. Sobolev solved this equation in the limit that the interaction region is infinitely narrow → "The Sobolev Approximation".

$$\phi(\nu - \nu_0) \equiv \delta(\nu - \nu_0)$$

and the size of the interaction region reduces to a point, the Sobolev point that can be derived from:

$$\nu_0 = \nu_p\,(1 - v_z(r_s)/c)$$

$$v_z(r_S) = \frac{z}{r_S}\,v(r_S) = c\left(1 - \frac{\nu_0}{\nu_p}\right)$$

The Sobolev optical depth

The optical depth at the frequency ν_p along a line in the direction z is defined as

$$\tau_{\nu_p}(z_1) = \int_{z_1}^{\infty} \kappa_{\nu_p}(z)\,\rho(z)\,dz = \frac{\pi\, e^2}{m_e\, c}\, f_l \int_{z_1}^{\infty} n_l(z) \left(1 - \frac{n_u\, g_l}{n_l\, g_u}\right) \phi(\Delta\nu) dz$$

since $\Delta\nu(z) = \nu_p\,(1 - v_z/c) - \nu_0$

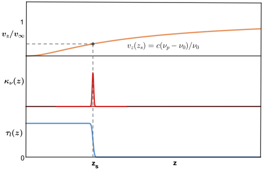
$$\tau_{\nu_p}(z_1) = \frac{\pi\, e^2}{m_e\, c}\, f_l \int_{\Delta\nu(z_1)}^{\Delta\nu)(z=\infty)} n_l(z) \left(1 - \frac{n_u\, g_l}{n_l\, g_u}\right) \frac{dz}{d(\Delta\nu)}\, \phi(\Delta\nu) d(\Delta\nu),$$

Assuming a delta function for the profile and integrating, we have,

$$\tau_{\nu_p}(z_1) = \frac{\pi\, e^2}{m_e\, c}\, f_l\; n_l(r_s) \left(1 - \frac{n_u\, g_l}{n_l\, g_u}\right) \left(\frac{dz}{d\Delta\nu}\right)_{r_s}$$

$$\boxed{\tau_{\nu_p}(z_1) = \chi_0^{\ell}(r_s) \left(\frac{dz}{d\Delta\nu}\right)_{r_s}} \rightarrow \text{Sobolev optical depth}$$

where χ_0^{ℓ} and $dz/d\Delta\nu$ are now evaluated at the Sobolev point.



The Sobolev optical depth is a step function

Calculation of $dz/d\Delta\nu$

Using the relationship between z and $\Delta\nu$

$$\Delta\nu = \nu_p \left(1 - \frac{z}{r}\frac{v(r)}{c}\right) - \nu_0 \qquad \text{where} \qquad r^2 = z^2 + p^2$$

$$\left(\frac{dz}{d\Delta\nu}\right)_{r_s} = \frac{c/\nu_0}{\mu^2 \frac{dv}{dr} + (1-\mu^2)\frac{v}{r}}$$

where $\mu = \cos\theta = z/r$ and $\nu_p \approx \nu_0$

$$\boxed{\tau_{\nu_0}(r_s) = \chi_o^\ell(r_s)\,\frac{c/\nu_0}{\mu^2 \frac{dv}{dr} + (1-\mu^2)\frac{v}{r}}}$$

The Sobolev optical depth in the radial direction ($\mu = 1$)

$$\tau_{\nu_0}(r_s, \mu = 1) = \chi_o^\ell(r_s)\left(\frac{dv}{dr}\right)^{-1} \quad \text{due to Doppler shift!}$$

The Sobolev optical depth in the tangential direction ($\mu = 0$)

$$\tau_{\nu_0}(r_s, \mu = 0) = \chi_o^\ell(r_s)\left(\frac{v}{r}\right)^{-1} \quad \text{due to spherical divergence!}$$

Conditions for the Sobolev approximation

Photons from the photosphere interact only with the gas at r_s and a narrow volume around it.

Narrow "line interaction region" are obtained if

- The absorption profile is narrow
- The velocity law is steep $\rightarrow$ large gradient dv/dr

Under this condition, the optical depth will also be reduced to a point, the "Sobolev point", which depends only on the local conditions!

The radiative transfer equations are simplified enormously for a moving medium.

- The fraction of radiation that reaches r_s is called "penetration probability"
- The radiation emitted or scattered from r_s is expressed in terms of an "escape probability"

The Penetration Probability

To compute the radiation field, we use the formal solution of the radiative transfer for the monochromatic intensity evaluated at $\tau_s(\nu, \mu)$,

$$I_\nu(\tau_\nu, \mu) = I_\nu^c\, e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(\tau_\nu)\, e^{-(\tau_\nu - t_\nu)}\, dt_\nu \qquad (1)$$

where I_ν^c is the radiation field from the star's photosphere, and

$$\tau_\nu = \int_{R_\star}^{r} \chi_0^\ell\, \phi(\Delta\nu)\, dz$$

$$\tau_{\nu_p} = \chi_0^\ell\, \frac{dz}{d\Delta\nu} \int_{\Delta\nu(R_\star)}^{\Delta\nu(r)} \phi\,(\Delta\nu)\; d(\Delta\nu) = \tau_0\, \Phi(\Delta\nu_\mu)$$

$$\Delta\nu_\mu = \nu_p - \nu_0\,(1 + \mu\, V/c)$$

$$I_{\nu_p}(\mu) = I^c_{\nu_0}\, e^{-\tau_0\, \Phi(\Delta\nu_\mu)}$$

$$\int_{\Delta\nu(R_\star)}^{\Delta\nu(r)} \phi\,(\Delta\nu)\ d(\Delta\nu) = \Phi(\Delta\nu_\mu)$$

Let's compute the mean intensity.

$$J_{\nu_p} = \frac{1}{2} \int_{\mu_\star}^{1} I_{\nu_p}(\mu) d\mu = \frac{1}{2}\, I^c_{\nu_0} \int_{\mu_\star}^{1} e^{-\tau_0\, \Phi(\Delta\nu_\mu)} d\mu$$

Integrating over all the profile

$$\bar{J}_{\nu_p} = \frac{1}{2}\, I^c_{\nu_0} \int_{\mu_\star}^{1} \int_{-\infty}^{\infty} \Phi(\Delta\nu_\mu) e^{-\tau_0\, \Phi(\Delta\nu_\mu)} d\mu\, d\Delta\nu_\mu$$

$$\bar{J}_{\nu_p} = \frac{1}{2}\, I^c_{\nu_0} \int_{\mu_\star}^{1} \int_{0}^{1} e^{-\tau_0\, d\Phi(\Delta\nu_\mu)}\, d\mu$$

$$\bar{J}_{\nu_p} = \frac{1}{2}\, I^c_{\nu_0} \int_{\mu_\star}^{1} \frac{1 - e^{-\tau_0}}{\tau_0}\, d\mu$$

$$\beta_c\, I^c_{\nu_0}$$

$$\beta_c = \frac{1}{2} \int_{\mu_\star}^{1} \frac{1 - e^{-\tau_0}}{\tau_0}\, d\mu$$

Is the penetration probability
For photons in a radial direction,

$$\beta_c = \frac{1 - \mu_\star}{2}\, \frac{1 - e^{-\tau_0}}{\tau_0}$$

where the Sobolev optical depth (when $\mu = 1$) is

$$\tau_{\nu_0}(r_s, \mu = 1) = \chi^\ell_o(r_s)\, \frac{c}{\nu_0} \left(\frac{dv}{dr}\right)^{-1}$$

Under the Sobolev approximation, the radiative acceleration takes a straightforward form,

$$\bar{g}^L_{rad} = \frac{\chi^\ell_0}{c} \oint \int_{-\infty}^{\infty} \phi(\Delta\nu)\, I^c_\nu \; e^{-\tau_0 \Phi(\Delta\nu_\mu)}\, d(\Delta\nu_\mu)\, \bar{n}\, d\Omega_{\bar{n}}$$

$I^c_{\nu_0}$ varies very slowly with the frequency, and the profile function $\phi(\Delta\nu_\mu)$ is very narrow.

$$g^L_{rad} = \frac{\chi^\ell_0}{c} \oint I^c_{\nu_0}(r,\Omega)\, \bar{n}\, d\Omega \int_{-\infty}^{\infty} e^{-\tau_0 \Phi(\Delta\nu_\mu)}\, d\Phi(\Delta\nu_\mu)$$

Finally,

$$\bar{g}^L_{rad} = \frac{\chi^\ell_0}{c} \oint \left[\frac{1 - e^{-\tau_0}}{\tau_0}\right] I^c(\Omega)\, \bar{n}\, d\Omega_{\bar{n}}.$$

For $\mu = 1$

$$g^L_{rad} = \frac{\chi^\ell_0 v_{th}}{c^2} \left(\frac{\nu_0 L_\nu}{L_*}\right) \frac{L_*}{4\pi r^2} \left[\frac{1 - e^{-\tau_0}}{\tau_0}\right]$$

Optical thin lines $\rightarrow \tau_0 \ll 1$

$$g_{rad}^{L} = \frac{\chi^L v_{th}}{c^2} \left(\frac{\nu_0 L_\nu}{L_*} \right) \frac{L_*}{4\pi r^2} \left[\frac{1 - e^{-\tau_0}}{\tau_0} \right]$$

$e^{-\tau_0} = 1 - \tau_0$

$$g_{rad}^{L}(thin) = \frac{\chi^L v_{th}}{c^2} \frac{L_*}{4\pi r^2} \quad (3)$$

Since it has a dependence $\propto 1/r^2$, similar to gravity acceleration, it leads to an apparent gravity (g_{eff}), or effective gravity:

$$g_{eff} = g + g_{rad}^{S} + g_{rad}^{L}(thin) = -\frac{GM_*(1 - \Gamma - \Gamma_{thin})}{r^2} \quad (4)$$

where

$$\Gamma_{thin} = \frac{N_{thin}\, \chi^L\, v_{th}\, L_\star}{4\pi c^2\, G\, M_\star}, \quad (5)$$

N_{thin} is the number of optically thin lines. The contribution of these thin lines leads to an effective gravity.

Remember to correct the gravity if you want to calculate the star's mass!

Optical thick lines $\rightarrow \tau_0 \gg 1$

The term inside the brackets tends to $\tau_0^{-1} = \kappa^L \, \rho \, L_S^{-1}$, so the line acceleration can be approximated by

$$g_{rad}^L(thick) = \frac{\chi^L(\bar{r}) v_{th}}{c^2} \frac{L}{4\pi r^2} \frac{1}{\tau_0} = \frac{L}{4\pi r^2 \rho \, c^2} \left(\frac{dv}{dr}\right)_{r_s}^{-1} . \qquad (6)$$

which depends on the velocity gradient.

Line acceleration for an ensemble of lines

The total line force, due to the addition of **all the single lines of atoms and ions** for a point star approximation and non-overlapping single lines, is given by:

$$g^{\mathrm{L}} = \sum_{\ell} \left(\frac{F_\nu \Delta\nu_D}{c} \right)_\ell \left(\frac{dv/dr}{\rho\, v_{\mathrm{th}}} \right) (1 - e^{-\xi^\ell t}).$$

The line acceleration can be expressed in terms of the Thomson acceleration

$$g^L_{rad} = \frac{\sigma_e\, F}{c} M(t)$$

where

$$M(t) = \sum_\ell \frac{\Delta\nu_D\, F_\nu}{F}\, \frac{1}{t}\, (1 - e^{-\xi^\ell t})$$

$$t = \frac{\sigma_e\, \rho\, v_{th}}{dv/dr} \quad \text{with} \quad \tau_0 = \xi^\ell t$$

All lines are in resonance with photons. The acceleration depends on the velocity gradient. Therefore, there is a non-linear feedback.

The multiplier parameters

Abbott (1982) approximated the force multiplier function, calculated with thousands of lines with an analytical approximation using three line-force parameters k, α, and δ.

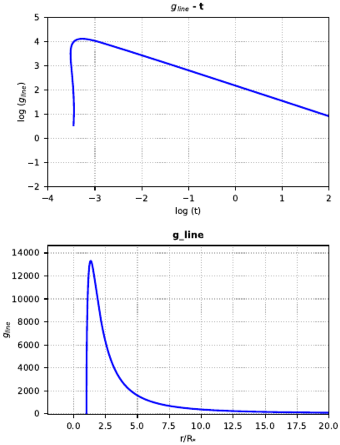
$$M(t) = k\, t^{-\alpha}\, (10^{-11}\, \frac{n_e}{W})^{\delta}$$

where k, and α are parameters determined by a power-law fit of t.

Since

$$t = \frac{\sigma_e\, \rho\, v_{th}}{dv/dr}$$

$$g^L_{rad} \propto (dv/dr)^{\alpha}$$



Typical values of the parameters

Each wind model is completely characterised by the four model parameters T_{eff}, $\log g$, n_e/W, and t

- $0 \leq \alpha \leq 1$
- Optically thick lines have $\alpha = 1$
- Optically thin lines have $\alpha = 0$
- Typical values $0.4 \leq \alpha \leq 0.6$

For O-type stars, $k \sim 0.1$.

- Often $\delta < 0.12$.
- For strong changes in the ionization equilibrium $\delta > 0.2$ or even $\delta < 0$ (Puls 2008).

The parameter k can be interpreted as the fraction of photospheric flux, which would be blocked if all lines were optically thick, divided by α.

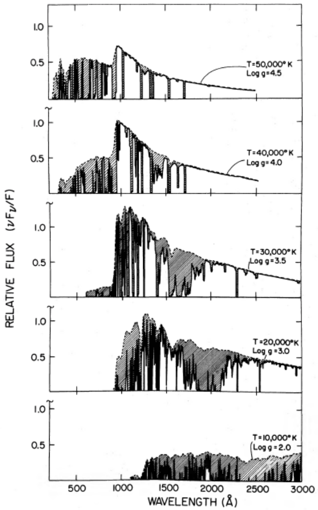


Figure 9: Emergent flux from the indicated model atmosphere of Kurucz (1979). Shaded areas show wavelengths where the momentum of the radiation field is absorbed by the wind. Extracted from **Abbott1982**.

Dependence with Metallicity

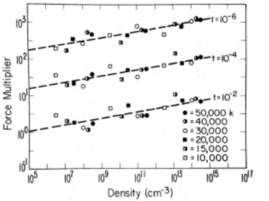
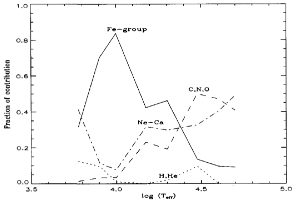


Figure 10: The dependence of the line acceleration on the density (n_e/W) of the absorbing gas in the wind for three representative values of t. Extracted from **Abbott1982**.



The calculated acceleration can be approximated by

$$\boxed{M(t) = 0.28\, t^{-0.56} (N_{11})^{0.99} \,(Z/Z_o)^{0.44}}$$

N_{11} in units of 10^{11} cm^{-3}

Z/Z_o is the mass fraction of metals relative to the sun.

The **line acceleration increases** with increasing metallicity and electron density.

Summary

- **Thomson acceleration**
  - $g^T \propto \frac{1}{r^2}$ for electron scattering
- **Radiative line acceleration**
  - $g_{rad}^L \propto \frac{1}{r^2}$ for optically thin lines
  - $g_{rad}^L \propto \frac{dv}{dr}$ for optically thick lines
  - $g_{rad}^L \propto \left(\frac{dv}{dr}\right)^\alpha$ for ensemble of lines
  - g_{rad}^L increases with metallicity

} **Doppler-effect** is important!

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Conclusions

The Sobolev approximation is

- A effective methods for modelling emission spectra
- It plays an important role in the radiation hydrodynamics

The present formulation for the line acceleration is valid for

- Spherically symmetric stellar wind
- Point radiation source
- Velocity laws that increase monotonically with radius
- Line source function approximated by the Sobolev theory.
- Completely non-coherent scattering

For regions close to the photosphere, where the size of the stellar disk is appreciable, the line acceleration of the line can be reduced up 40%.

The effects of line overlap are the largest source of uncertainty. Generally, it is

- overestimated at small radii, and
- underestimated at large radii

A different parameterisation has been suggested by Gayley (1995), and both parameterisations are consistent though.