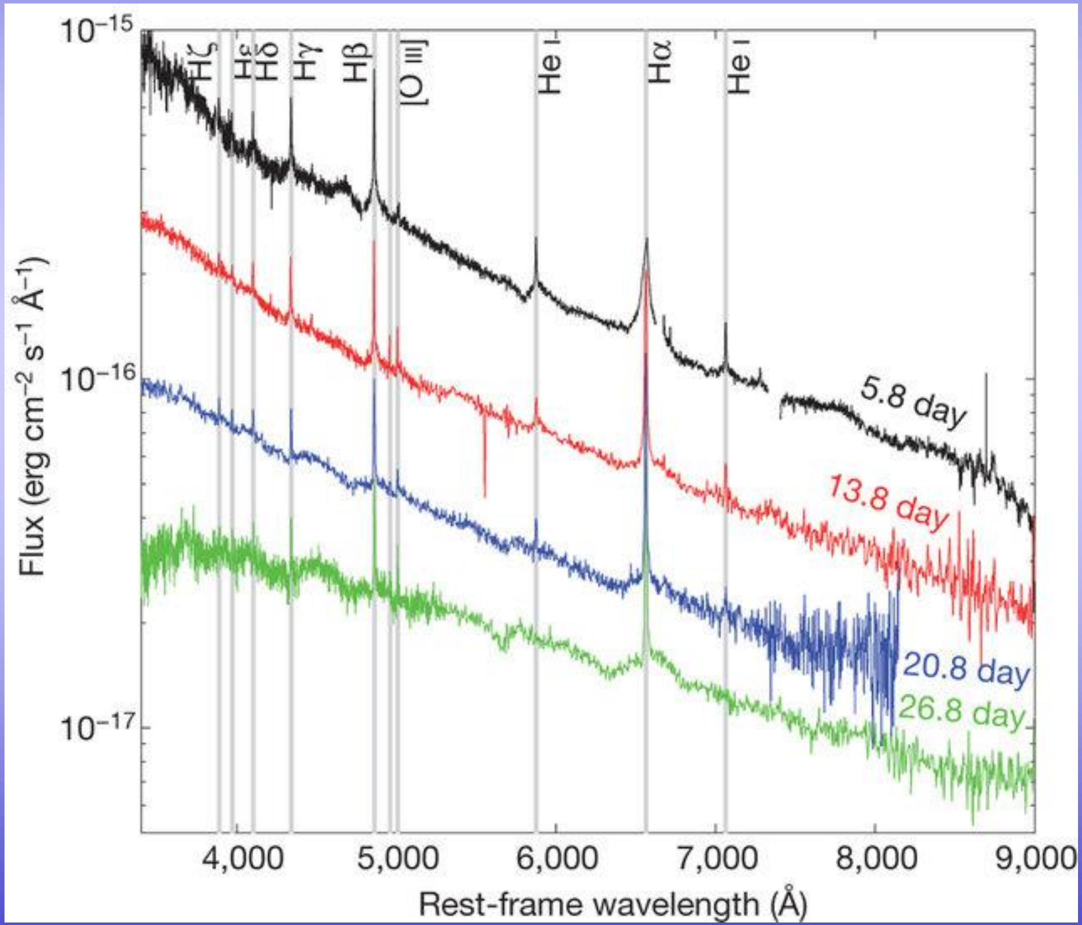
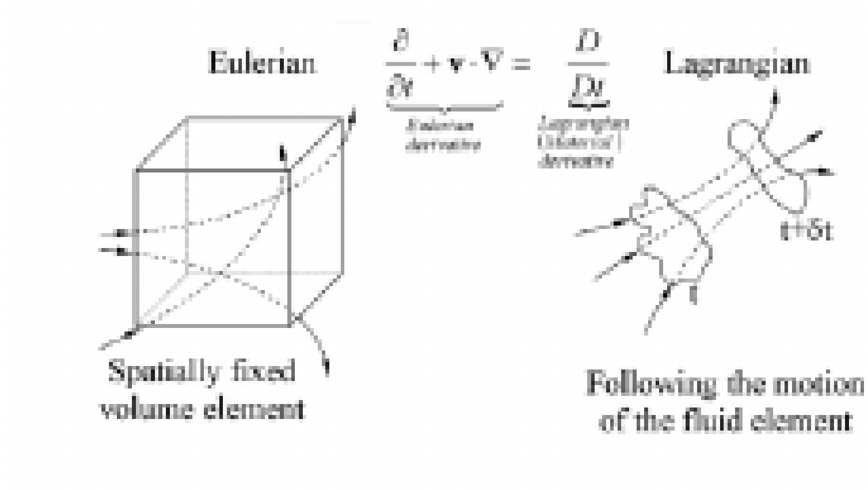
Radiative Transfer in Moving media



The Laboratory Frame

The observer and the center of symmetry of the star are at rest. The calculation of the scattering terms limits the maximum velocity to a few times the mean thermal velocity.



The Comoving Frame (CMF)

The interaction partner is at rest

$$\nu_0 = \gamma\,\nu\,(1 - \bar{\beta}\cdot\bar{n})$$

This induce aberration

$$\bar{n}_0 = \frac{\nu}{\nu_0}\left[\bar{n} - \left(1 - \frac{\gamma\,\bar{\beta}\cdot\bar{n})}{(\gamma+1)}\right)\,\gamma\,\bar{\beta}\right]$$

$\bar{\beta} = \frac{\bar{v}}{c}$ and $\gamma = (1-\beta^2)^{-1/2}$

$$\frac{\partial I}{\partial X} = \frac{\partial I}{\partial X}\Big|_{\nu 0} + \frac{\partial I}{\partial \nu}\frac{\partial \nu}{\partial X}$$

The comoving frame of the fluid

Advantages

- η and χ coefficients are isotropic
- The scattering integral can be calculated assuming an angle-averaged redistribution function.
- Complete redistribution can be assumed
- Line formation problems in the presence of large flow-velocity gradients are greatly facilitated when working in the comoving frame of the fluid

Disadvantages

- The radiative transfer equation in the CMF contains a term involving the frequency derivative of the intensity. it is an initial-plus-boundary-value problem for coupled partial integro-differential equations.
- A separate calculation is necessary to obtain the emergent radiation field in the frame of a stationary observer.

The Radiative Transfer Equation in the CMF.

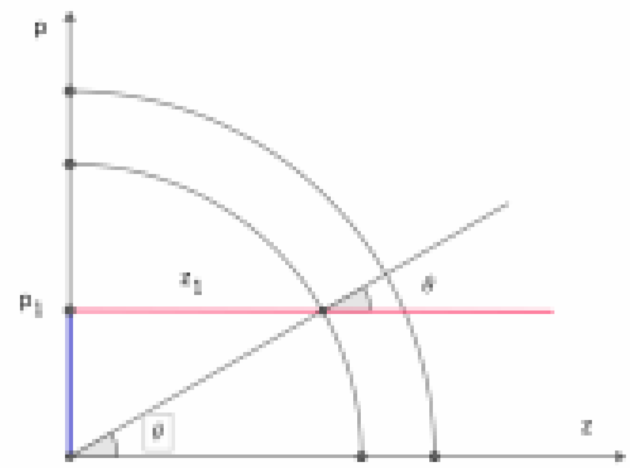
The transfer equation for radiation of frequency ν, flowing in a direction $\mu = \cos(\theta)$ with the radius vector r as seen by an observer moving with a gas element, in a spherically symmetric configuration with radial flow velocities $v(r)$, is

$$\mu \frac{\partial}{\partial r} I(\nu,\mu,r) + \frac{1-\mu^2}{r}\frac{\partial}{\partial \mu} I(\nu,\mu,r) - \frac{\alpha}{r}\left\{1-\mu^2+\beta\,\mu^2\right\}\frac{\partial}{\partial \nu} I(\nu,\mu,r)$$

$$= \eta(\nu,r) - \chi(\nu,r) I(\nu,\mu,r)$$

In the CMF, the total emissivity (η) and opacity (χ) are isotropic.

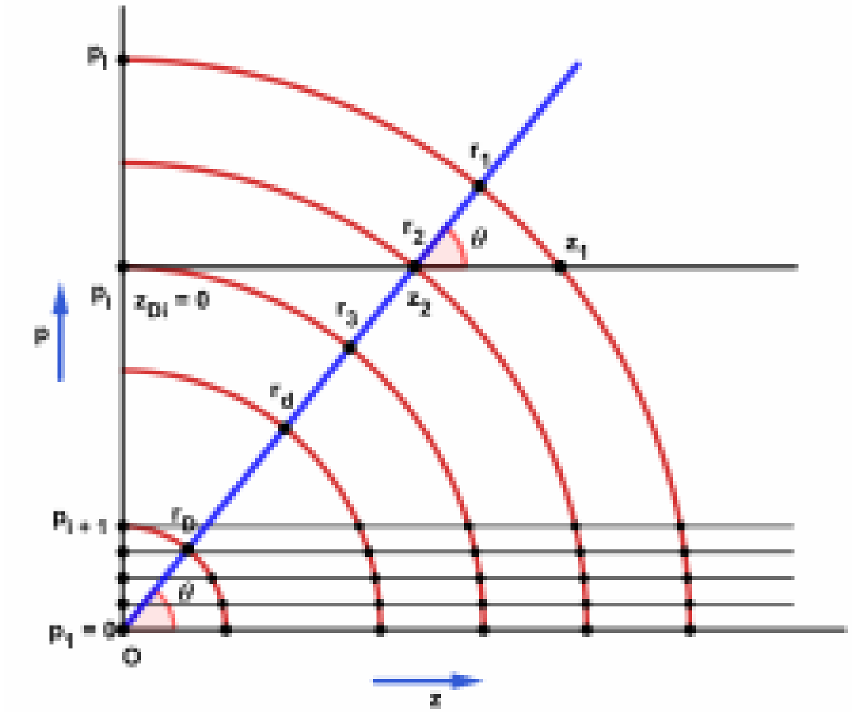
$$\alpha(r) \equiv \nu_0\, v(r)/c,$$
$$\beta(r) \equiv d\ln v(r)/d\ln r.$$



The impact parameters

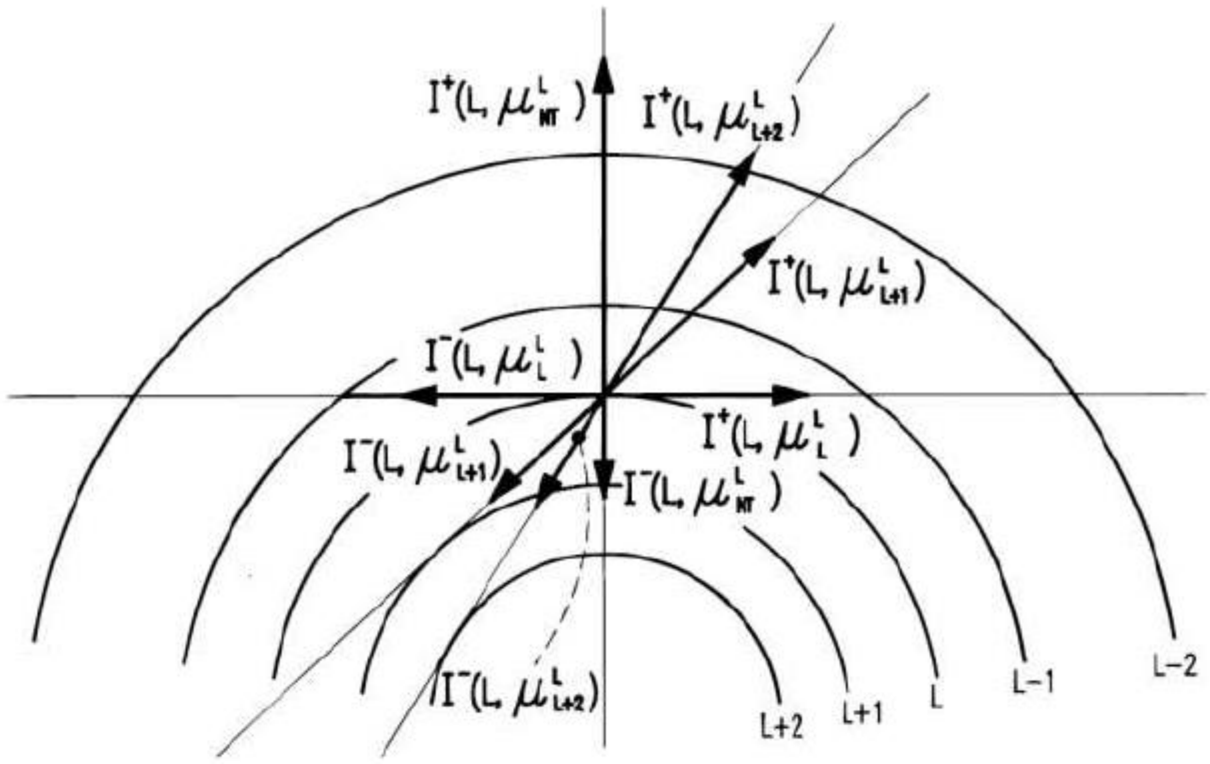
It is helpful to use a coordinate system (z, p), specified by a set of parallel rays parametrised by the perpendicular distance to the center of symmetry

- p: the impact parameter
- z: the distance z along the ray
- $r = \left(p^2 + z^2\right)^{1/2}$



The RT of frequency ν flowing along a ray with impact parameter p is

$$\pm \frac{\partial}{\partial z} I^{\pm}(\nu, p, z) - \frac{\alpha(r)}{r}\left[1 - \mu^2 + \beta(r)\mu^2\right]\frac{\partial}{\partial \nu} I^{\pm}(\nu, p, z) = \eta(\nu, r) - \chi(\nu, r) I^{\pm}(\nu, p, z),$$



Set of specific intensity selected for the calculation of the mean intensity (Gros et al. 1997)

For each ray p, we can define I^+ and I^- *so that*

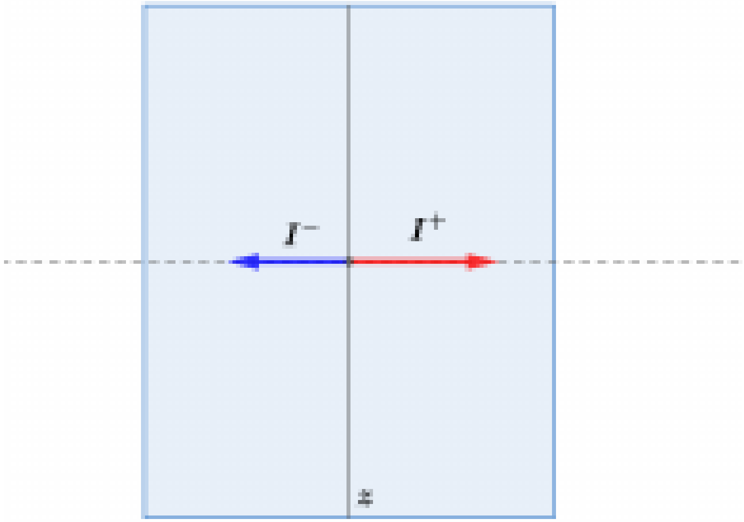
$$\pm \frac{\partial}{\partial z} I_\nu^\pm(p,z) - \gamma_\nu(r) \frac{\partial}{\partial \nu} I_\nu^\pm(p,z) = \eta_\nu(r) - \chi_\nu(r) I_\nu^\pm(p,z),$$

with $\gamma_\nu(z) \equiv \frac{\alpha(r)}{r\chi_\nu(r)} \left[1 - \mu^2 + \beta(r)\mu^2\right]$

We can also define a "mean intensity" and "average flux" along p

$$U_\nu(p,z) \equiv \frac{1}{2}\left[I_\nu^+(p,z) + I_\nu^-(p,z)\right]$$

$$V_\nu(p,z) \equiv \tfrac{1}{2}\left[I_\nu^+(p,z) - I_\nu^-(p,z)\right].$$



$$\frac{1}{\chi_\nu(r)} \frac{\partial}{\partial z} U_\nu(z) - \gamma_\nu(z) \frac{\partial}{\partial \nu} V_\nu(z) = -V_\nu(z)$$

$$\frac{1}{\chi_\nu(r)} \frac{\partial}{\partial z} V_\nu(z) - \gamma_\nu(z) \frac{\partial}{\partial \nu} U_\nu(z) = S_\nu(z) - U_\nu(z)$$

$S_\nu(r) = \eta_\nu(r)/\chi_\nu(r)$ $\rightarrow$
The source function

The total opacity and emissivity at frequency ν, evaluated in the comoving frame, can be written as

$$\chi_\nu(r) = \chi^L(r)\phi(\nu, r) + \chi^c(r) + \sigma_e n_e(r)$$

$$\eta_\nu(r) = \eta^L(r)\phi(\nu, r) + \eta^c(r) + \sigma_e n_e(r) J^c(r),$$

L: line and C: continuum processes

The opacity and emissivity in the $l \rightarrow u$ transition depends on the NLTE occupational numbers (n_l and n_u), which are obtained via the rates equations.

$$\chi_{lu}(\nu) = \sigma_{lu}(\nu)\left[n_l - g_{lu}(\nu) n_u\right],$$

$$\eta_{lu}(\nu) = \left(2h\nu^3/c^2\right)\sigma_{lu}(\nu) g_{lu}(\nu) n_u$$

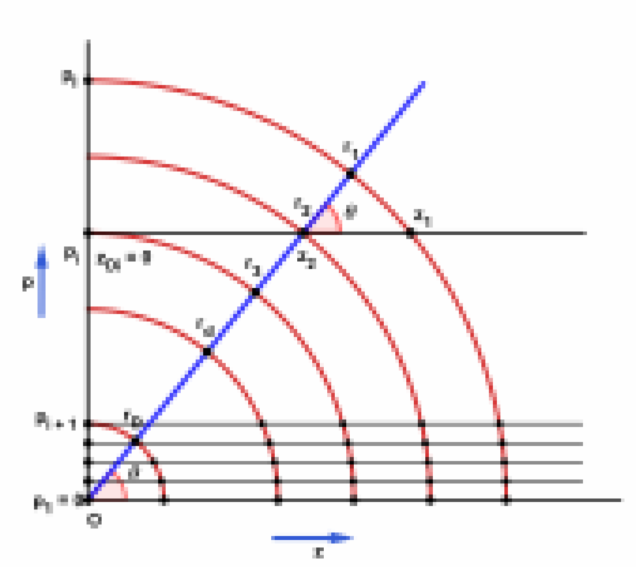
The total source function is

$$S_\nu \equiv \frac{\eta^L(r)\phi(\nu, r) + \eta^c(r) + \sigma_e n_e(r) J^c(r)}{\chi^L(r)\phi(\nu, r) + \chi^c(r) + \sigma_e n_e(r)} = \zeta_\nu(r)\bar{J}(r) + \Theta_\nu(r)$$

$$\bar{J}(r) = \int_{-\infty}^{\infty} d\nu\, \phi_\nu(r) \int_0^1 d\mu\, U_\nu(p, z).$$

Boundary and initial conditions

Our system of equations requires the specification of both **boundary and initial conditions**.



Spatial outer boundary conditions

- $r = R \qquad I^{-}(\nu, p, z_{\max}) = 0$ then
  - $V_\nu(p, z_{\max}) = U_\nu(p, z_{\max})$
    $z_{\max} = \left(R^2 - p^2\right)^{1/2}$

$$\frac{1}{\chi(\nu, R)} \frac{\partial}{\partial z} u(\nu, z_{\max}) = \gamma(\nu, z_{\max}) \frac{\partial}{\partial \nu} u(\nu, z_{\max}) - u(\nu, z_{\max}) .$$

Inner boundary conditions ⇒

For $p > rc$ $I^{+}(p,0) = I^{-}(p,0)$ => $V(p,0) = 0$

For $p < rc$ $U(p, z_{min})$ => diffusion approximation

Difference equations

Difference the above equations

$$\frac{f_{k,d+1} q_{k,d+1} r^2_{D+1} J_{k,D+1}}{\Delta X_{kd} \Delta X_{k,d+1/2}} - \frac{f_{kd} q_{kd} r_d^2 J_{kd}}{\Delta X_{kd}} \left(\frac{1}{\Delta X_{k,d+1/2}} + \frac{1}{\Delta X_{k,d-1/2}} \right) + \frac{f_{k,d-1} q_{k,d-1} r^2_{d-1} J_{k,d-1}}{\Delta X_{kd} \Delta X_{k,d-1/2}}$$

and

$$\left(f_{k2} q_{k2} r_2^2 J_k^2 - f_{k1} q_{k1} r_1^2 J_{k1} \right) / \Delta X_{k,3/2} = r_1^2 h_k J_{k1}.$$

The Eddington factors $f_\nu, g_\nu, h_\nu, n_\nu$, and the sphericity factor q_ν are given. In practice, these factors are evaluated by means of a ray-by-ray formal solution in the comoving frame. The Source function is given.

Similarly, for a continuum transition,

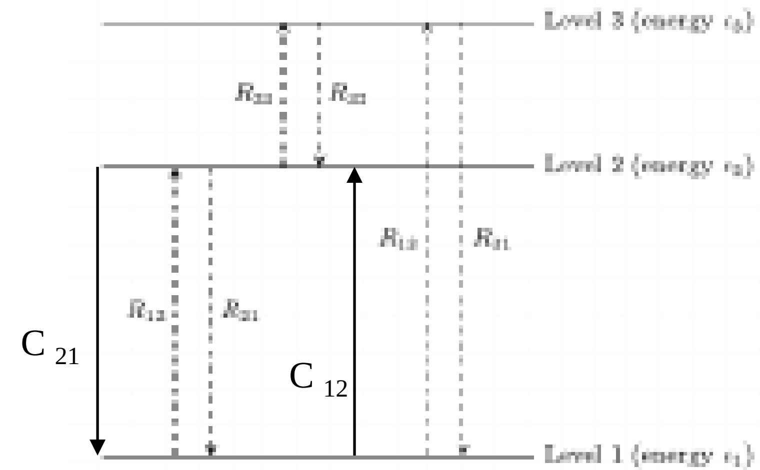
$$S_{lu,kd} = a_{lu,kd} \sum_{k'} \frac{4\pi w_{k'} \sigma_{lu,k'} J_{k'd}}{h\nu_{k'}} + b_{lu,kd} + c_{lu,kd} J_{kd},$$

where the coefficients a, b, and c are given by equations (4.19)-(4.21) with $\gamma_{lu,d}$ and $\epsilon_{lu,d}$ replaced by $\gamma_{lu,kd}{}'$ and $\epsilon_{lu,kd}{}'$, respectively. The summation covers the frequency range (v_{lu0}, v_{lu1}).

The Equivalent Two-atom Approach

An expression for (n_l/n_u) can be obtained directly from rows l and u of the statistical equilibrium equations.

where $P_{ij} \equiv R_{ij} + C_{ij}$.



$$n_l \left(R_{lu} + \sum_{i<l} P_{li} + \sum_{l<j\neq u} P_{lj} + C_{lu} \right) - n_u P_{ul} = \sum_{i<l} n_i P_{il} + \sum_{l<j\neq u} n_j P_{jl}$$

where

$$-n_l P_{lu} + n_u \left(R_{ul} + \sum_{u>i\neq l} P_{ui} + \sum_{j>u} P_{uj} + C_{ul} \right) = \sum_{u>i\neq l} n_i P_{iu} + \sum_{j>u} n_j P_{ju},$$

$$S_{lu}(\nu) = \gamma_{lu}'(\nu) \int_{\nu_{0lu}}^{\nu_{1lu}} \frac{4\pi\sigma_{lu}(\nu) J_\nu}{h\nu} d\nu + \epsilon_{lu}'(\nu),$$

Explicit transtitions

$$n_l(R_{lu} + a_1) - n_u P_{ul} = a_2$$

$$-n_l P_{lu} + n_u(R_{ul} + a_3) = a_4$$

$$(n_l/n_u) = (R_{ul} + \alpha_{lu})/(R_{lu} + \beta_{lu})$$

$$S_{lu} = [\bar{J}_{lu} + (\beta_{lu}/B_{lu})]/[1 + (\alpha_{lu} - g_{lu}\beta_{lu})/A_{ul}] \equiv \gamma_{lu}] \equiv \gamma_{lu}\bar{J}_{lu} + \epsilon_{lu}$$

System of algebraic equations for the ocupational numbers

$$\mathscr{A}_d \boldsymbol{n}^d = \mathscr{B}_d \,,$$

$$\boldsymbol{n}^d \equiv (n_{1d}, \ldots, n_{Ld}, n_{\alpha d}, \ldots, n_{\omega d}, n_{kd})^T$$

$$Z_{td} \equiv n_{ld} R_{lu,d} - n_{ud} R_{ul,d}$$

$$\delta \boldsymbol{n}^d = \sum_t \frac{\partial \boldsymbol{n}^d}{\partial Z_{td}} \delta Z_{td} \qquad \boldsymbol{\delta n}_l = \sum_t \mathscr{N}_{lt} \boldsymbol{\delta Z}_t$$

$$\boldsymbol{\delta Z}_t = \mathscr{L}_t \boldsymbol{\delta n}_l + \mathscr{U}_t \boldsymbol{\delta n}_u + \mathscr{R}_t$$

$$\mathscr{M}_t \boldsymbol{\delta Z}_t \equiv (\boldsymbol{I} - \mathscr{L}_t \mathscr{N}_{lt} - \mathscr{U}_t \mathscr{N}_{ut}) \boldsymbol{\delta Z}_t = \mathscr{L}_t \left(\sum_{t' \neq t} \mathscr{N}_{lt'} \boldsymbol{\delta Z}_{t'} \right) + \mathscr{U}_t \left(\sum_{t' \neq t} \mathscr{N}_{ut'} \boldsymbol{\delta Z}_{t'} \right) + \mathscr{R}_t \,.$$

Lambda Iteration Method

Build an atmospheric + wind model

Compute absorption and emission coefficients in LTE

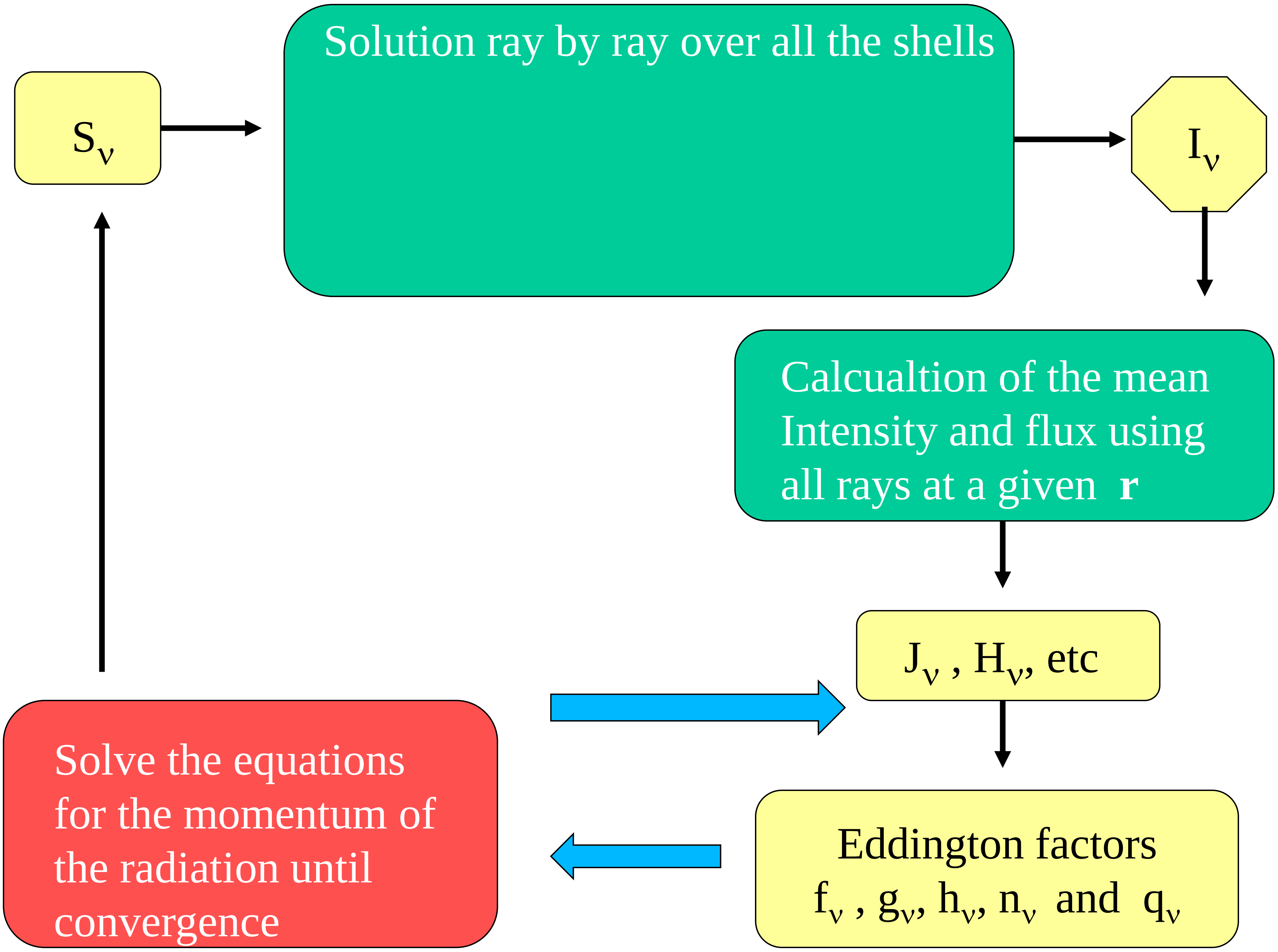
Solve the radiative transfer (RT) equation for the continuum

$$f_\nu = K_\nu / J_\nu, \qquad g_\nu = N_\nu / H_\nu$$

Solve the RT equations for explicit lines
dI/dν is initial condition (blue wind)

Rate equations → Iterate until n_u/n_l reaches convergence -- ETLA scheme

Solve the RT equations for selected continua and lines.



S_ν
Solution ray by ray over all the shells
I_ν
Calcualtion of the mean Intensity and flux using all rays at a given **r**
J_ν , H_ν, etc
Eddington factors
f_ν , g_ν, h_ν, n_ν and q_ν
Solve the equations for the momentum of the radiation until convergence

Once the problema is solved we have S_ν and I_ν

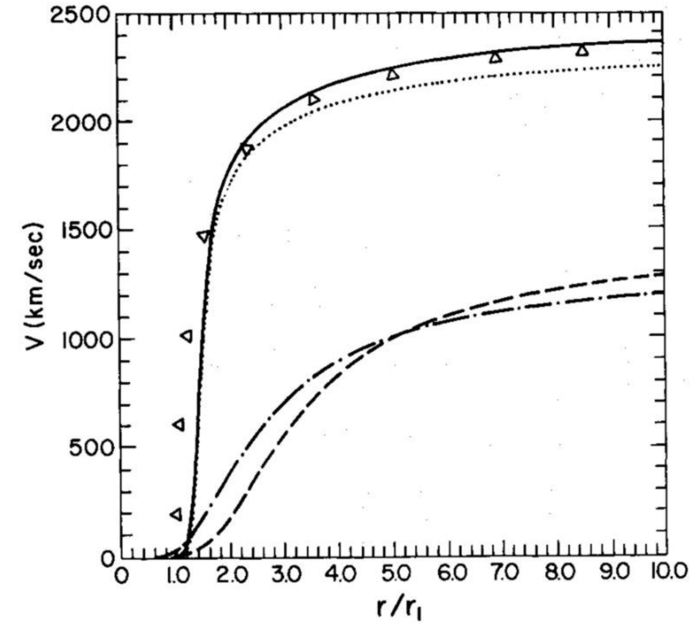
J_ν, K_ν, H_ν are given in the atom's frame for the lines and the continuun radiation

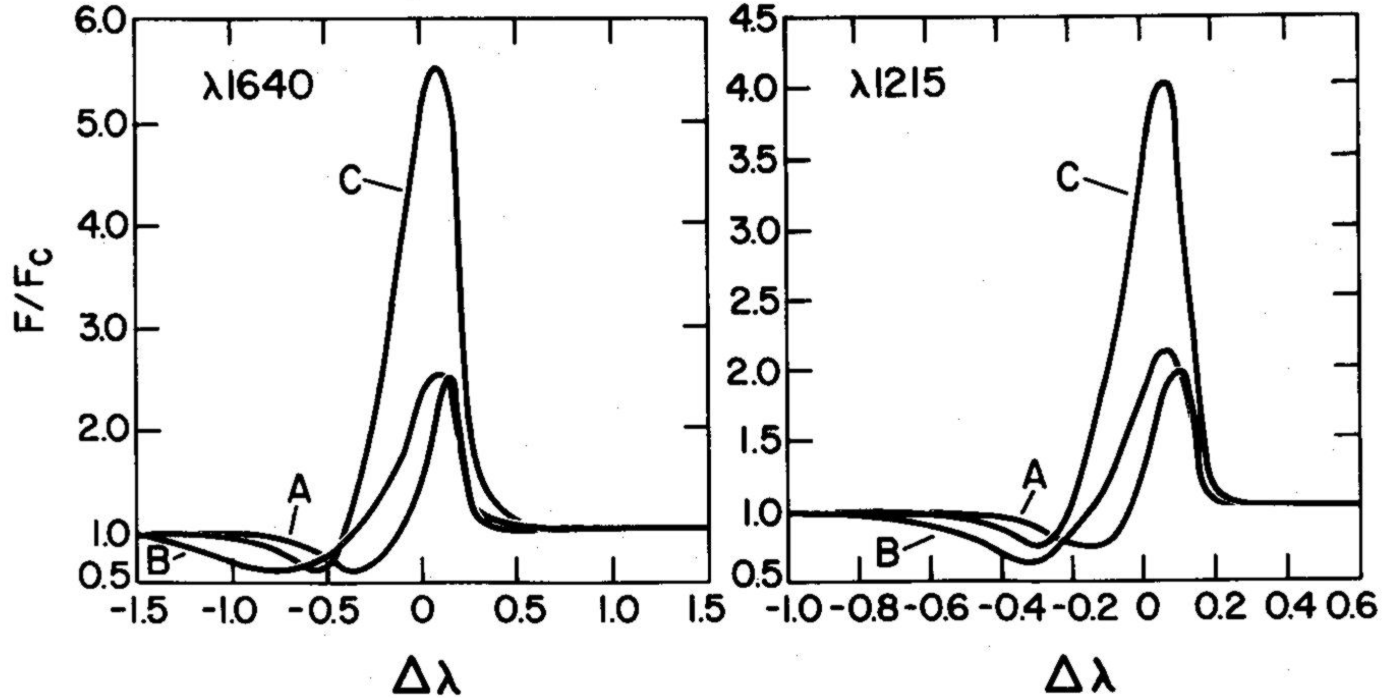
Then, we have to solve the TR equation to calculate the spectrum in the observer's frame.

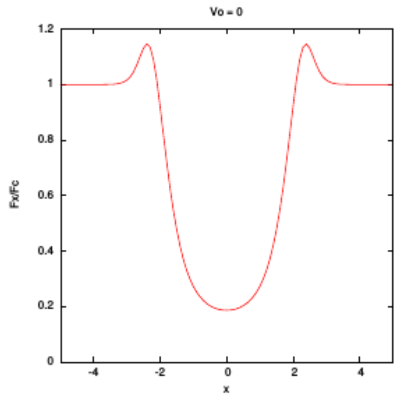
This last step is solved using the **known**

source function S_ν for the lines (Doppler effect).

Examples of line calculations
Mihalas et al. (1978)







$$V(r) = V_0\left(\frac{r_c}{r}\right)^{\alpha}$$

$$\chi_c = \chi_0\left(\frac{r_0}{r}\right)^{2-\alpha}$$

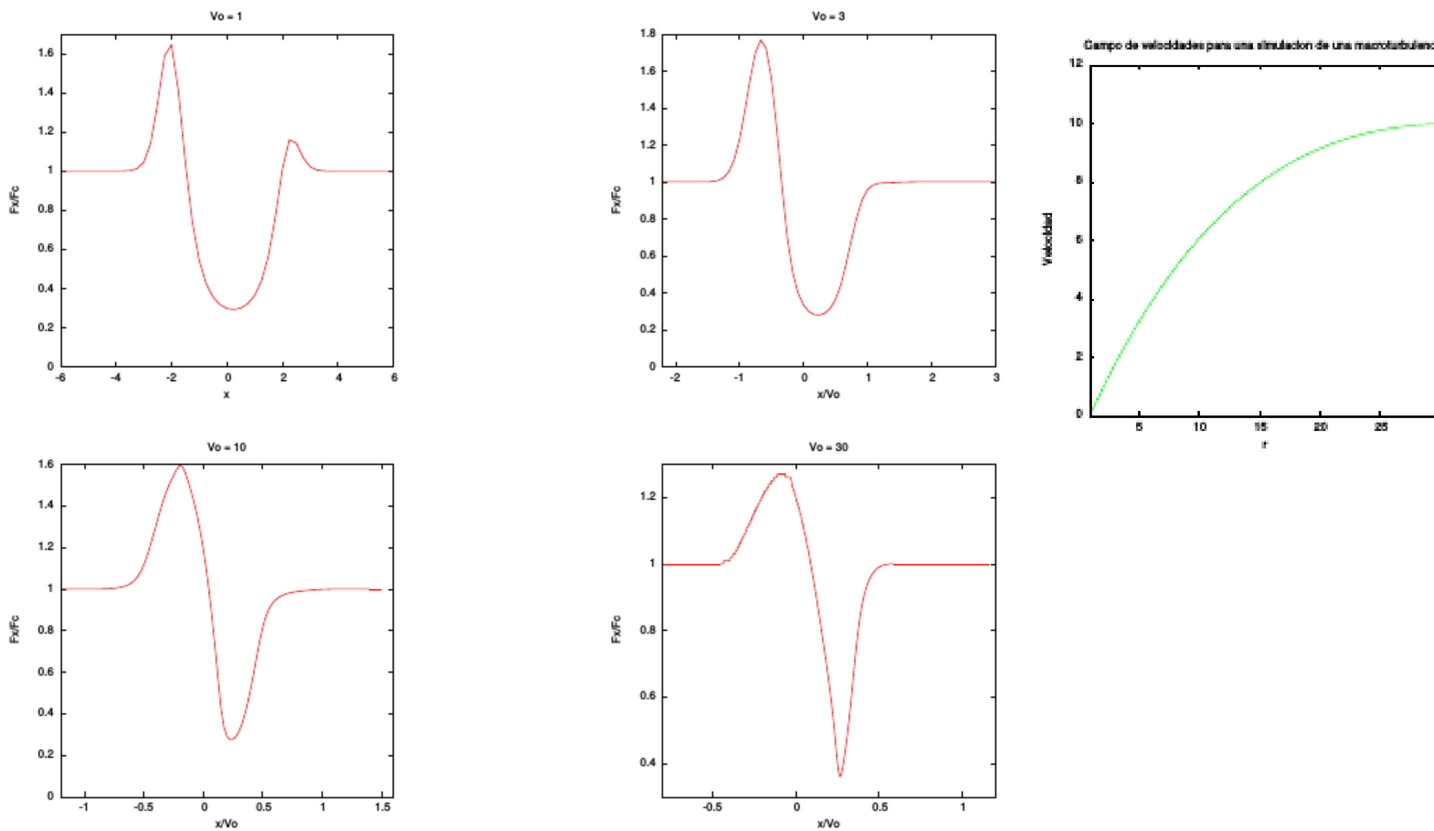
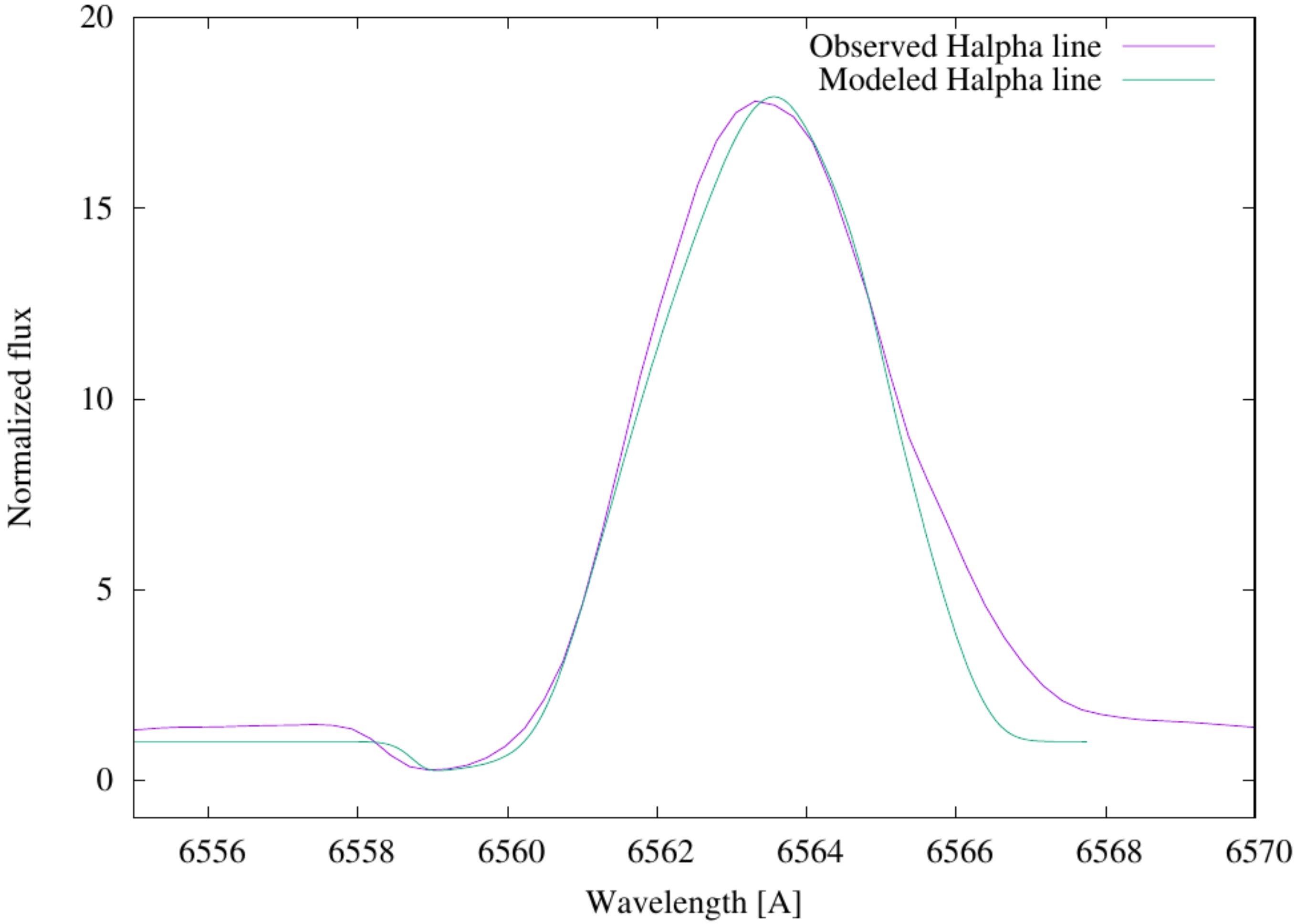
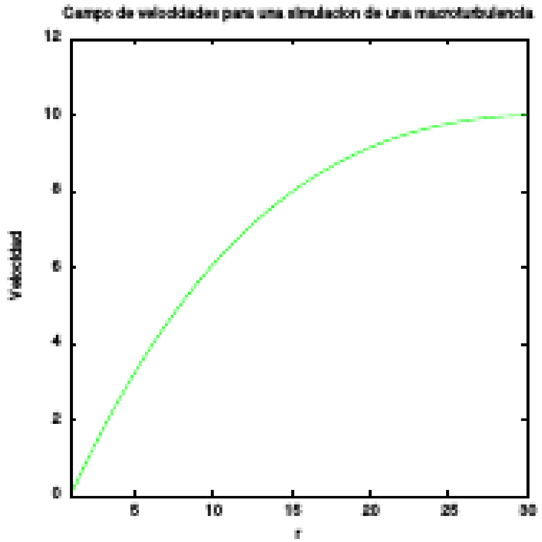


Figura 6.5: Flujos emergentes obtenidos para una atmósfera con las mismas propiedades físicas que las consideradas en Mihalas (1980).

PCygni star modelled using a velocity beta-law



Numerical Methods: Pros and Cons



The Lagrange reference frame → Comoving frame

- Good performance for **large velocity fields**
- Opacity and emissivity coefficients are isotropic
- Complete redistribution functions
- Line profiles are symmetric (small interval of frequencies)

- It can be used only with monotonic velocity field
- Coefficients are vectors and Matrixes

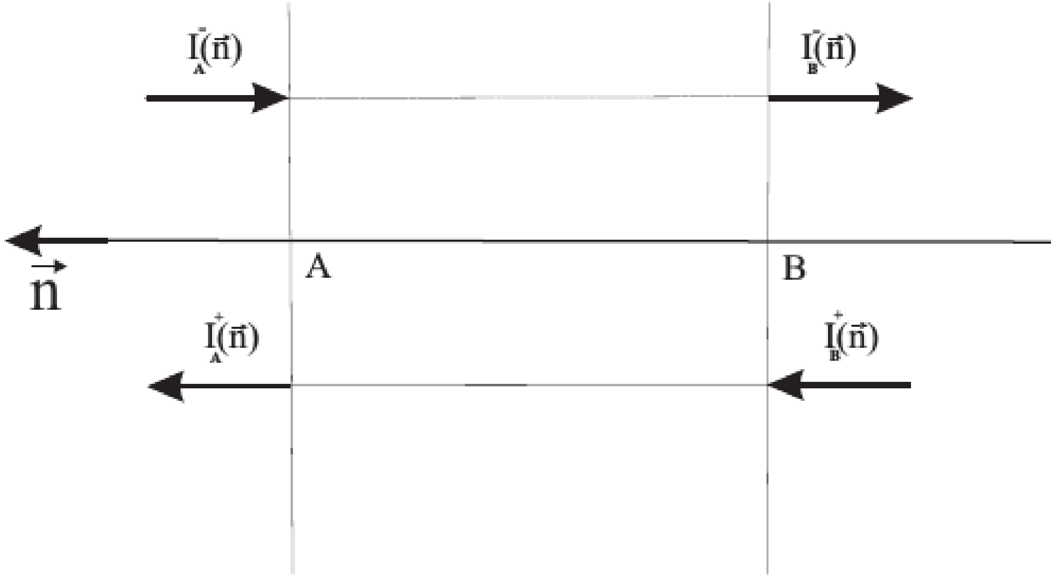
Implicit Integral Method (IIM)

Simonneau & Crivelari (1993)

Forth & Back Implicit Lambda Iteration -- FBLIT
By Atanackovic-Vukmanovic et al. (1997)

$$\pm\mu\frac{d}{d\tau}I^{\pm}(\tau,\mu,\nu)=\varphi^{\pm}(\tau,\mu,\nu)\left[I^{\pm}(\tau,\mu,\nu)-S^{\pm}(\tau,\mu,\nu)\right]$$

$$\tau^{\pm}(\tau,\mu,\nu)\equiv\int_0^{\tau}\varphi^{\pm}(t,\mu,\nu)dt$$



Plane-parallel atmosphere. Intensity for incoming and outcoming directions

The incoming and outcoming directions

$$I^-(\tau,\mu,\nu) = I_A^-(0,\mu,\nu)\, e^{-\tau^-(\tau,\mu,\nu)/\mu} + \int_0^{\tau^-(\tau,\mu,\nu)} S^-(t,\mu,\nu)\, e^{-\left(\tau^-(\mu,\nu)-t\right)/\mu}\, dt/\mu$$

$$I^+(\tau,\mu,\nu) = I_B^+(\tau_B,\mu,\nu)\, e^{-\left(\tau_B(\mu,\nu)-\tau^+(\tau,\mu,\nu)\right)/\mu} + \int_{\tau^+(\tau,\mu,\nu)}^{\tau_B(\mu,\nu)} S^+(t,\mu,\nu)\, e^{-\left(t-\tau^+(\tau,\mu,\nu)\right)/\mu}\, dt/\mu$$

$$J_\varphi(\tau) = \frac{1}{2}\int_0^\infty d\nu \int_{-1}^{+1} d\mu\, \varphi(\tau,\mu,\nu) I(\tau,\mu,\nu),$$

$$S_\nu(\tau) = f_{1\nu}(\tau) + f_{2\nu}(\tau) \sum_{L=1}^{NL} \sigma_L(\tau) J_\varphi(\tau_L)$$

σ_L (τ) are polynomial that depends on τ

$$
\begin{aligned}
I^{+}(\tau,\mu,\nu) &= I_B^{+}(\tau_B,\mu,\nu)\, e^{-\left(\tau_B(\mu,\nu)-\tau^{+}(\tau,\mu,\nu)\right)/\mu} \\
&+ \int_{\tau^{+}(\tau,\mu,\nu)}^{\tau_B(\mu,\nu)} S^{+}(t,\mu,\nu)\, e^{-\left(t-\tau^{+}(\tau,\mu,\nu)\right)/\mu}\, dt/\mu
\end{aligned}
$$

$$
S_\nu(\tau) = f_{1\nu}(\tau) + f_{2\nu}(\tau) \sum_{L=1}^{NL} \sigma_L(\tau) J_\varphi(\tau_L)
$$

We introduce S in the TR equation. We separate thermal terms form the scattering ones

$$
I_i^{-} = \gamma_i^{-} + \sum_{L=1}^{NL} \Lambda_{iL}^{-} J\varphi(\tau_L)
$$

$$
I_i^{+} = \gamma_i^{+} + \sum_{L=1}^{NL} \Lambda_{iL}^{+} J\varphi(\tau_L)
$$

→ NL equations

NDxNF+1 coefficients. The coefficients are saved in each layer. This is the most important relation of the IIM

$$J_\varphi = \sum_{I=1}^{ND} \sum_{J=1}^{NF} A(\tau_L, \mu_I, \nu_J) I^+(\tau_{L+1}, \mu_I, \nu_J) + C(\tau_L)$$

$$\begin{aligned} I^-(\tau_L, \mu_I, \nu_J) &= \sum_{i=1}^{ND} \sum_{j=1}^{NF} R(\mu_I, \nu_J, \mu_i, \nu_j) I^+(\tau_{L+1}, \mu_i, \nu_j) + \alpha(\mu_I, \nu_J) \\ &+ \beta(\mu_I, \nu_J) J_\varphi(\tau_{L+1}) \end{aligned} \tag{3.32}$$

$$\begin{aligned} I^+(\tau_L; \mu_I, \nu_J) &= I^+(\tau_{L+1}; \mu_I, \nu_J) exp[-\Delta\tau(\mu_I, \nu_J)/\mu_I] + g_0(\mu_I, \nu_J) \\ &+ g_1(\mu_I, \nu_J) J_\varphi(\tau_L) + g_2(\mu_I, \nu_J) J_\varphi(\tau_{L+1}) \end{aligned} \tag{3.33}$$

S is expressed as a linear relationship

$$J_\varphi^-(\tau_L) = b_L^- S(\tau_L) + c_L^- S'(\tau_L),$$

$$\begin{aligned} I^-(\tau_{L+1}, \mu, \nu) &= I^-(\tau_L, \mu, \nu) e^{-\Delta\tau/\mu} + \gamma_1^- S(\tau_L, \mu, \nu) \\ &+ \gamma_2^- S(\tau_{L+1}, \mu, \nu) + \gamma_2'^- S'(\tau_{L+1}, \mu, \nu) \end{aligned}$$

$$I^-(\tau_{L+1}, \mu, \nu) = \mathcal{A}^-(\mu, \nu) + \mathcal{B}^-(\mu, \nu) S(\tau_{L+1}) + \mathcal{C}^-(\mu, \nu) S'(\tau_L + 1),$$

After integrating, we have

$$J_\varphi^-(\tau_{L+1}) = \hat{a}_{L+1}^- + \hat{b}_{L+1}^- S(\tau_{L+1}) + \hat{c}_{L+1}^- S'(\tau_{L+1}).$$

$$J_\varphi = \sum_{I=1}^{ND} \sum_{J=1}^{NF} A(\tau_L, \mu_I, \nu_J) I^+(\tau_{L+1}, \mu_I, \nu_J) + C(\tau_L)$$

Capa L=1

NDxNF+1 coefficients

Capa L= K-1

$$I^-(\tau_{L+1}, \mu, \nu) = \mathcal{A}^-(\mu, \nu) + \mathcal{B}^-(\mu, \nu) S(\tau_{L+1}) + \mathcal{C}^-(\mu, \nu) S'(\tau_L + 1),$$

Obtain S and I
the found relati

Capa L=K

$$\mathcal{A}^-(\mu, \nu) = I^-(\tau_L, \mu, \nu) e^{-\Delta\tau/\mu} + \gamma_1^- S^0(\tau_L, \mu, \nu)$$

Capa L=K+1

Compute coefficients

Capa L=NL

initial conditions

Radiative transfer equation in a moving media

$$\nu' = \nu - \xi^c(\vec{n} \cdot \vec{v}/c).$$

$$\mu \partial I(z, \mu, \nu)/\partial z = \eta(z, \mu, \nu) - \chi(z, \mu, \nu) I(z, \mu, \nu)$$

$$x_j \equiv \frac{\nu_j - \xi^c}{\bar{\Delta}\nu_D}.$$

$$x'_j \equiv x_j - \mu V.$$

$$\kappa(z, \mu, x) \equiv \kappa_c(z) + \kappa_l(z)\phi(z, \mu, x)$$

$$\eta(z, \mu, x) \equiv \eta_c(z) + \eta_l(z)\phi(z, \mu, x),$$

$$\phi(z, \mu, x) \equiv \phi(z, x - \mu V).$$

$$S(z,\mu,x) \equiv \left[\phi(z,\mu,x)S_l(z) + \beta_c(z)S_c(z)\right] / \left[\phi(z,\mu,x) + \beta_c(z)\right]$$

$$\frac{dI(z,\mu,x)}{d\tau(z,\mu,x)} = I(z,\mu,x) - S(z,\mu,x).$$

$$\begin{aligned} I(z_{max},\mu,x) &= I(0,\mu,x)e^{-\tau(0,\mu,x)} + \int_0^{\tau(0,\mu,x)} S(z,\mu,x)e^{-\tau(z,\mu,x)}d\tau(z,\mu,x) \\ &= I(0,\mu,x)e^{-\tau(0,\mu,x)} + \\ &+ \int_0^{\tau(0,\mu,x)} \left[\phi(\tau, \S_l(\tau) + \beta_c(\tau)S_c(\tau)\right] e^{-\tau}\kappa_l(\tau)d\tau \end{aligned}$$

$$S(\tau,\mu,x) = f_1(\tau,\mu,x) + f_2(\tau,\mu,x) \int_{-\infty}^{+\infty} dx \int_{-1}^{+1} I(\tau,\mu,x)\,\phi(\tau,\mu,x)\,d\mu$$

IIM is similar to Feautrier but use scalar operators instead of vectors and matrix

$I_L^+(J,I) \quad I_{L+1}^+(J,I)$

$$\begin{aligned} I_L^+(J,I) &= e^{-\Delta\tau^+(J,I)/\mu(I)} I_{L+1}^+(J,I) + G0(J,I) + G1(J,I)J_L \quad (4.13) \\ &+ G2(J,I)J_{L+1} + GP1(J,I)J'_L + GP2(J,I)J'_{L+1} \end{aligned}$$

$$\begin{aligned} I_L^-(J,I) &= \sum_{JP=1}^{NF}\sum_{IP=1}^{ND} R(J,I,JP,IP)I_{L+1}^+(JP,IP) + T(J,I) \quad (4.14) \\ &+ S1(J,I)J_L + S2(J,I)J_{L+1} + SP1(J,I)J'_L + SP2(J,I)J'_{L+1} \end{aligned}$$

$$\Delta\tau^\pm(J,I) = \frac{1}{\mu_I}\int_{\tau_L}^{\tau_L+1} (\beta_c(\tau) + \phi^\pm(\tau,\mu_I,x_J))d\tau$$

$$\int_{x_1}^{x_2} f(x)dx \approx \frac{\Delta x}{2}(f_1 + f_2) + \frac{\Delta x^2}{12}(f_1' - f_2') \tag{4.30}$$

$$\gamma(\tau)\Delta\tau = \Delta\nu^c \leq \frac{1}{2}.$$

$$\int_{\tau_1}^{\tau_2} \phi(\tau,\nu)d\tau \equiv \frac{\Delta\tau}{2}\left[[\phi(\tau_1,\nu) + \phi(\tau_2,\nu)] + \frac{\Delta\tau}{2}\left[\phi'(\tau_1,\nu) - \phi'(\tau_2,\nu)\right]\right] \tag{4.93}$$

$$x_J \equiv \frac{\nu_J - \xi^c}{\Delta \bar{\nu}_D} \qquad \phi_L(x_J, \mu_I) = \frac{\bar{\Delta}\nu_D}{\Delta\nu_{DL}} \phi(\bar{x}_J)$$

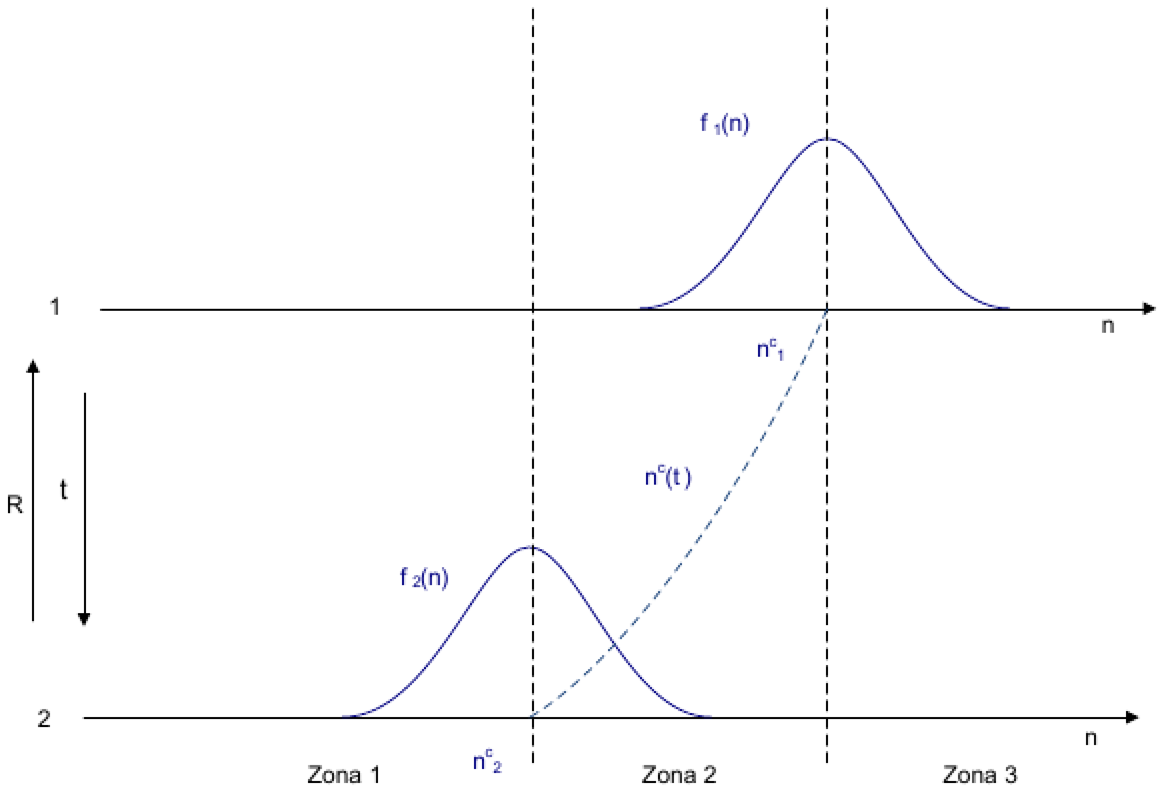
$$\begin{aligned}
[\frac{d}{d\tau}\phi(x_J, \mu_I)]_L &= [\frac{d}{d\tau}\frac{\Delta\bar{\nu}_D}{\Delta\nu_D}]\phi(\bar{x_J}) \\
&+ (\frac{\Delta\bar{\nu}_D}{\Delta\nu_D})_L \frac{d\phi(\bar{x_J})}{d\bar{x_J}}\{[\frac{d}{d\tau}(\frac{\Delta\bar{\nu}_D}{\Delta\nu_D})]_L(x_J - x_L^0) \\
&- (\frac{\Delta\bar{\nu}_D}{\Delta\nu_D})_L(\frac{dx^0}{d\tau})_L\}
\end{aligned}$$

In the local frame the profile is symmetric

$$\phi(\bar{x}) = \phi(-\bar{x})$$

In the observer's frame, it is asymmetric

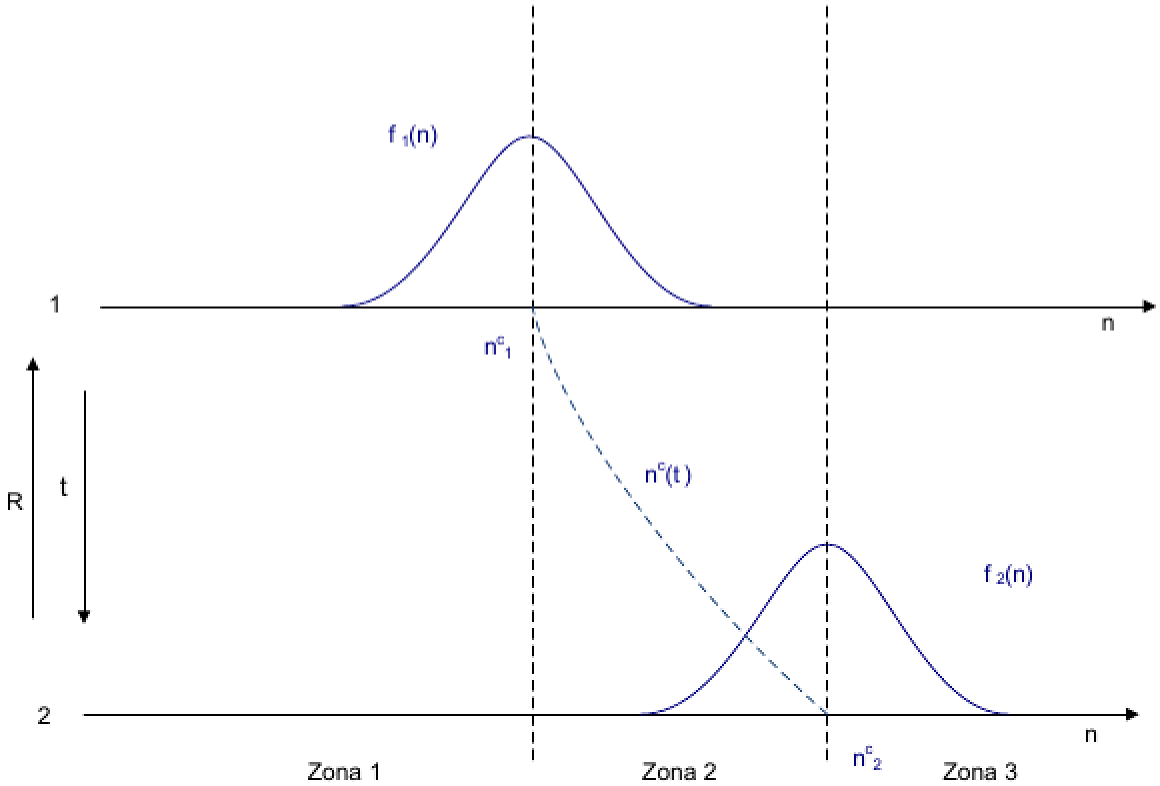
$$\phi(-x, -\mu) = \phi(x, \mu)$$



Shift of the line profile between two consecutive layers. Cases A and B

$\frac{dV}{d\tau} < 0$ y $\mu > 0.$

$\frac{dV}{d\tau} > 0$ y $\mu < 0$



Shift of the line profile between two consecutive layers. Cases C and D

$\frac{dV}{d\tau} < 0$ y $\mu < 0$.

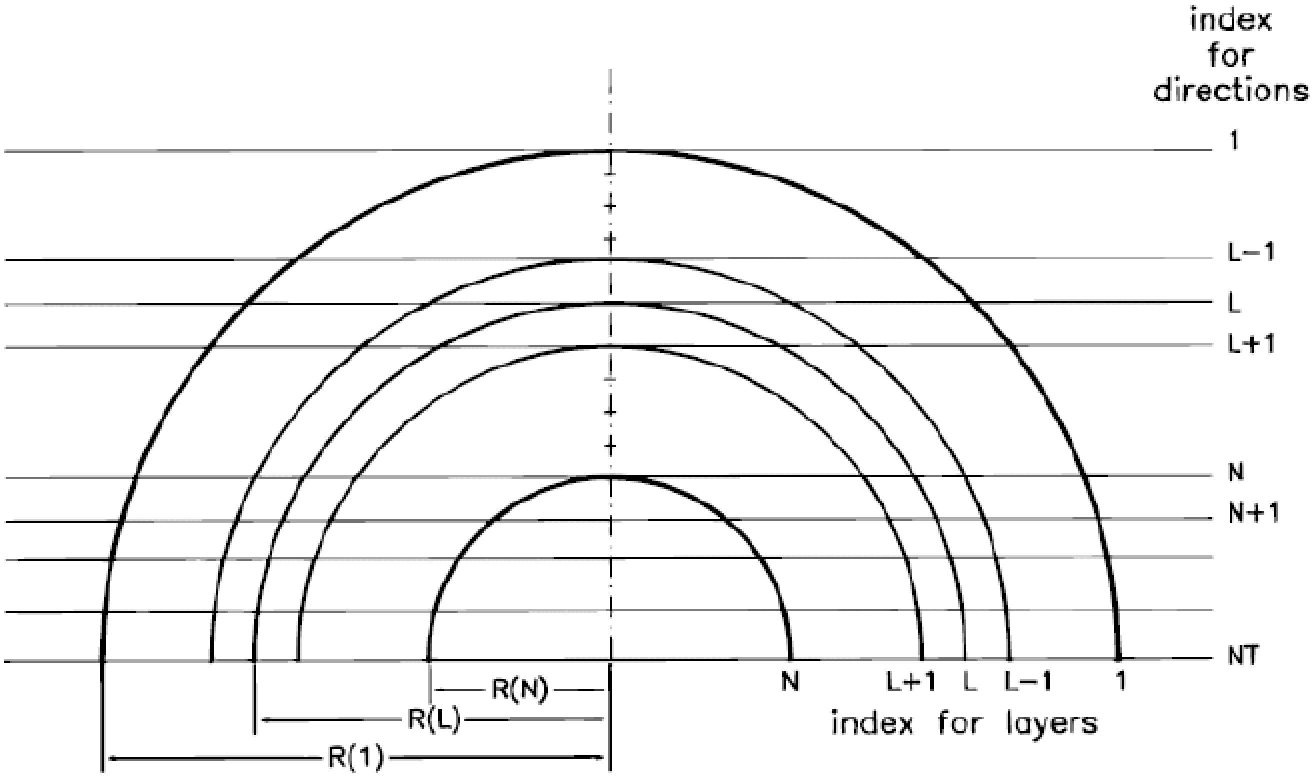
$\frac{dV}{d\tau} > 0$ y $\mu > 0$

$$\int_{\tau_1}^{\tau_2} \phi(\tau,\nu)d\tau \approx \frac{\Delta\tau}{2}\left[[\phi(\tau_1,\nu)+\phi(\tau_2,\nu)] + \frac{\Delta\tau}{2}[\phi'(\tau_1,\nu)-\phi'(\tau_2,\nu)]\right]$$

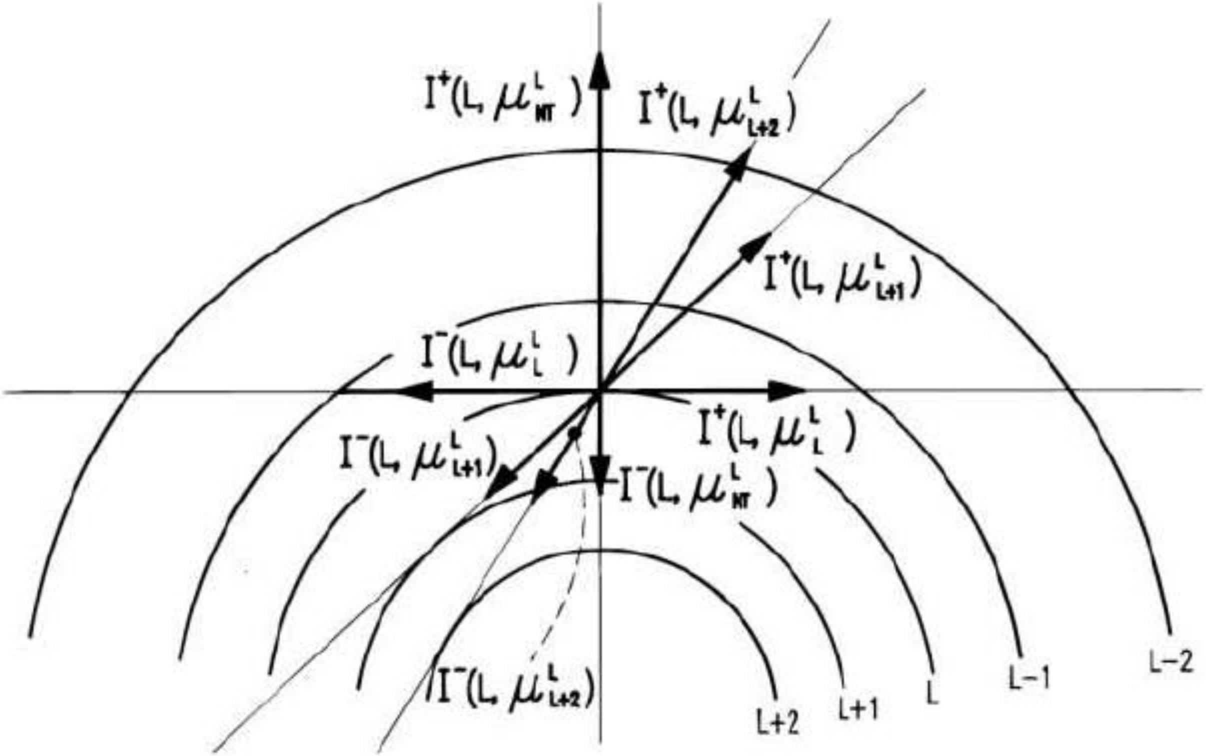
$$J_\phi \approx 2\sum_{J=1}^{NF} WF(J)\phi(x_J)\int_0^{+1} d\mu \frac{I^+(x_J,\mu)+I^-(x_J,\mu)}{2}$$

WF: the Gaussian quadrature
NF: frequency grid
$I^+(x, \mu) \neq I^-(x, \mu)$ the profile is not symmetric → all frequencies should be considered

The IIM for Moving Media



Grid of radii and impact parameters (Gros et al 1997)

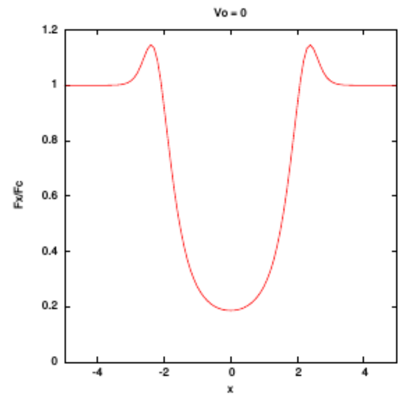


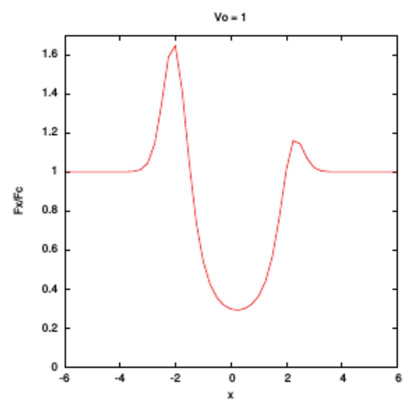
Set of specific intensity selected for the calculation of the mean intensity (Gros et al. 1997)

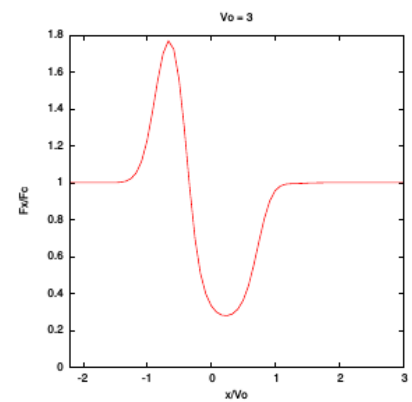
Shape of line profiles for different values of Vo

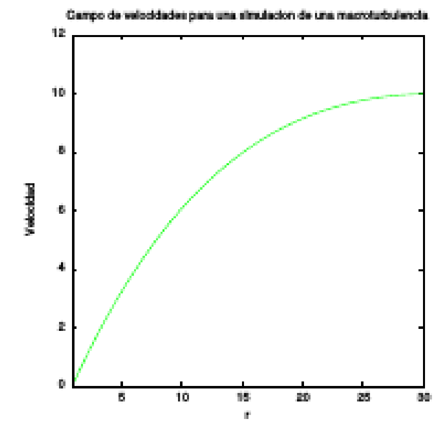
$$V(r) = V_0 \left(\frac{r_c}{r}\right)^{\alpha}$$

$$\chi_c = \chi_0 \left(\frac{r_0}{r}\right)^{2-\alpha}$$ opacity

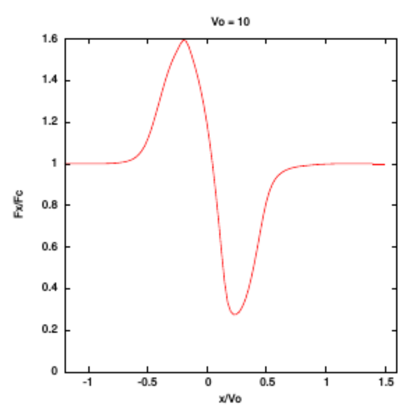








Velocity law



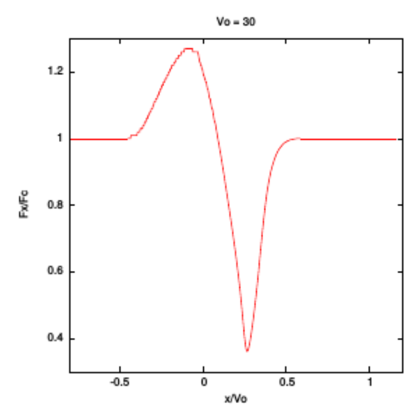
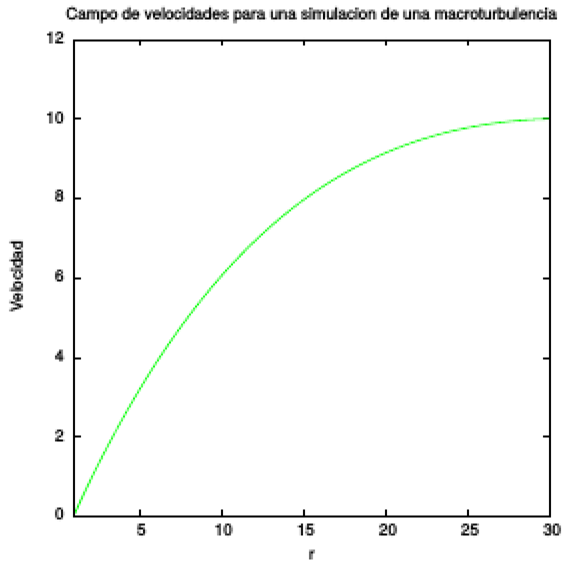
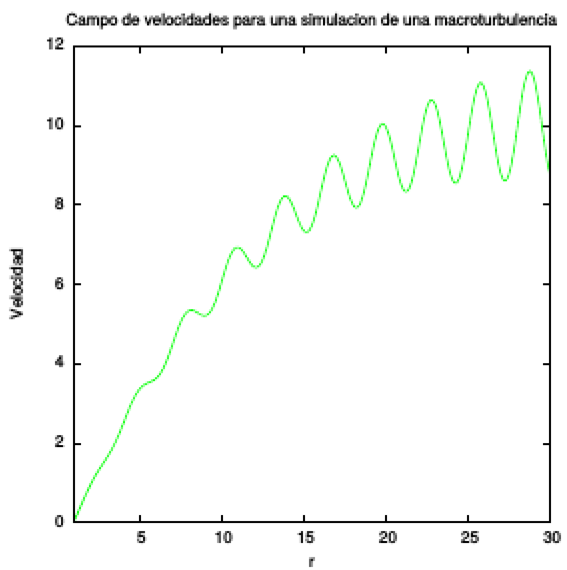


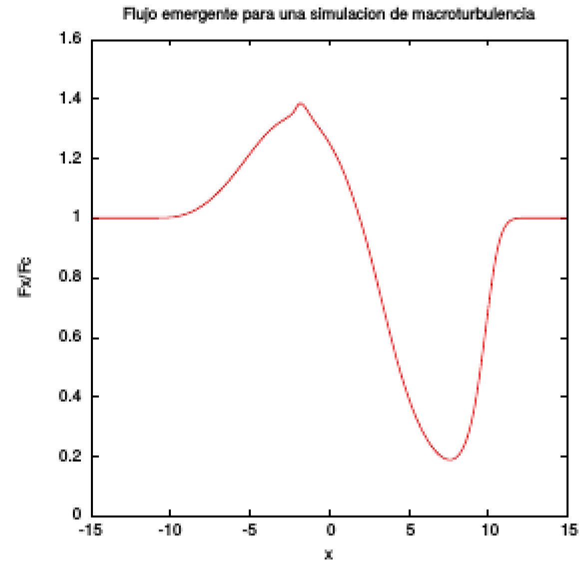
Figura 6.5: Flujos emergentes obtenidos para una atmósfera con las mismas propiedades físicas que las consideradas en Mihalas (1980).

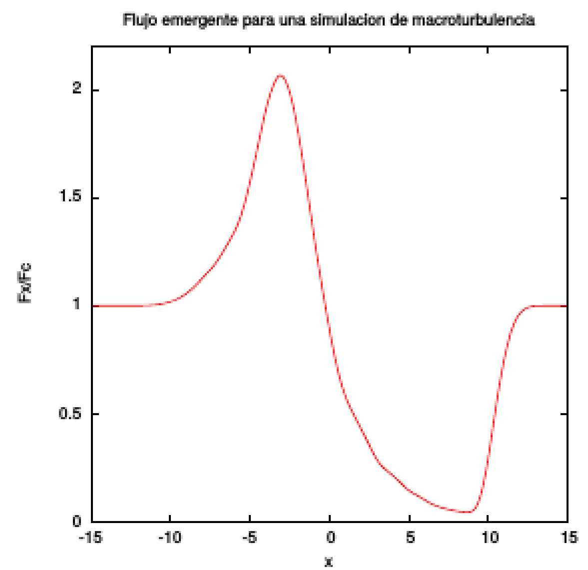
Velocity laws





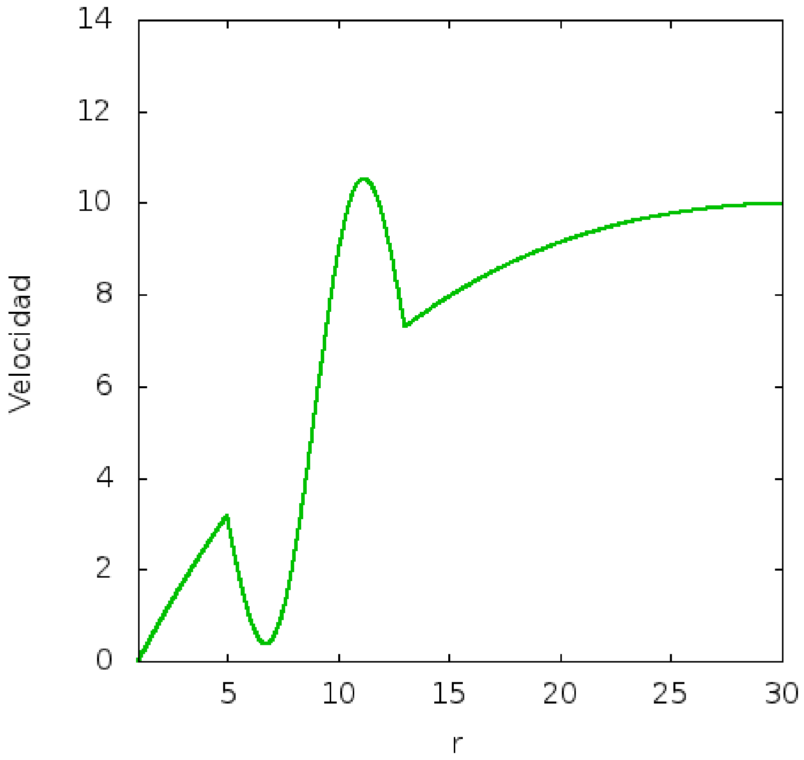
Line profiles



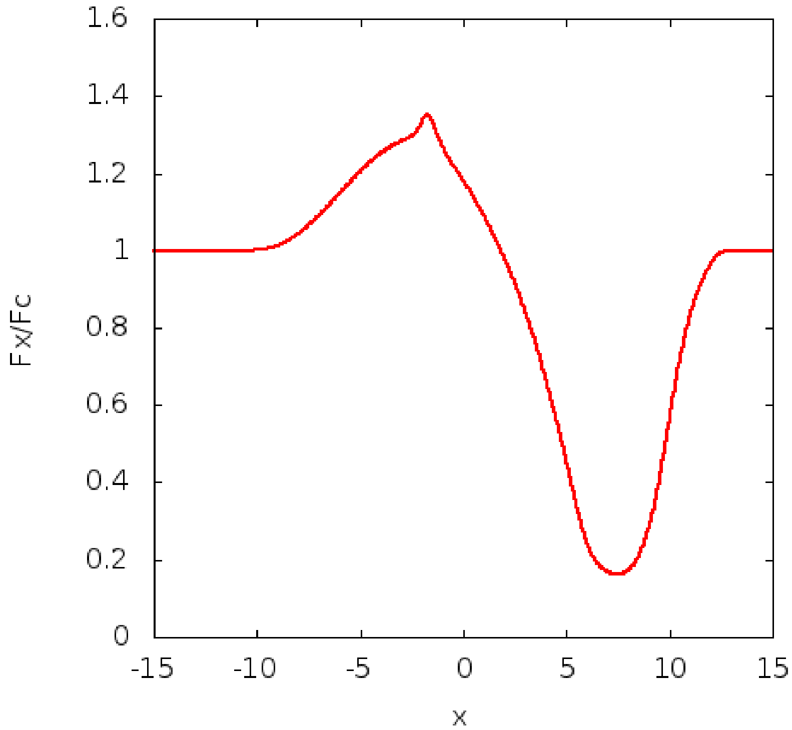


M. Colazo (2013) PhD thesis

Sinusoidal Pertubation



Effect on the line profile



M. Colazo (2013) PhD thesis

Numerical Methods: Pros and Cons

The Euler reference frame → IIM FBLIT

- It can be used with non monotonic velocity field
- Coefficients are scalar.
- Easy to implement
- Bad performance: high-time consuming process when **large velocity fields** are considered