NLTE modeling of polarized radiation in solar flares
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• resonance & impact polarization
  • radiation transfer
  • polarization in flares?

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Polarization in everyday life

Fishing:

LCD Displays:

Heidinger effect:

Dancing under polarized sky:

http://www.polarization.com
What is light polarization?

Classical image:
Electromagnetic waves are transversal → can be polarized in the plane perpendicular to the direction of propagation.

Quantum image:
Photons are particles with spin 1 oriented in the direction of propagation → statistical correlation of these spins may lead to measurable polarization.

basis composed of 2 base state vectors

x
y
z – quantization axis
Macroscopic quantitative description of polarized radiation

**UNPOLARIZED case:**

\[ I(\omega, \Omega) \]

**Polarized case:**

Specific intensity Stokes vector (parameters)

\[ S(\omega, \Omega) = (I, Q, U, V) \]

Mathematically: scalar calculated as squared norm of electric field. For non-monochromatic field: averaged over reasonable time \( \tau \) of measurement (physically must be \( \tau >> 10^{-8} \) s for common intensities)

Mathematically: composed of bilinear products of components of electric vector. For non-monochromatic field: averaged over reasonable time \( \tau \) of measurement (physically must be \( \tau >> 10^{-8} \) s for common intensities)
Atomic state description

What formalism is suitable?

1. Atom in pure state $|\psi\rangle$: description suitable for single atom

2. Statistical ensemble of atoms with “classical probabilities” of states $w_i$: description via density matrix operator (mixed state) $\rho = \sum_{j=1}^{N} w_i |\psi_i\rangle\langle\psi_i|$, $(\sum_{j=1}^{N} w_i = 1)$

Density matrix contains all the information about population of levels, their polarization and coherences between states.

The behavior of density matrix is driven by Schrödinger (Liouville) equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho]$$

where $H$ is Hamiltonian of the system, “[,]” is comutator.

In our approach: local electromagnetic field enter $H$ as independent parameter (atom in a “bath” of photons).

The approximate solution of the Schrödinger equation can be found using perturbation series…
How is radiation scattered in the frame of QED? (2-level atom)

Coherent scattering:

Incoherent scattering (2 absorptions):

Energy conservation law leads to restrictions on scattered photons. In 2-photon process only the total energy must be conserved:

\[ 2\omega_s = \omega_A + \omega_B \]
Two-level atom, normal Zeeman triplet, anisotropic irradiation

Irradiation by anisotropic unpolarized radiation leads to Zeeman sublevels population imbalance. We say that upper level is polarized.

Irradiation in direction of quantization axis populates $M_U = -1$ and $M_U = +1$ Zeeman sublevels of upper level.

The density matrix can be used to characterize upper level population and polarization:

$$
\rho_0^0 = \frac{1}{\sqrt{3}} (n_{M=-1} + n_{M=0} + n_{M=+1})
$$

$$
\rho_0^2 = \frac{1}{\sqrt{6}} (n_{M=-1} - 2n_{M=0} + n_{M=+1})
$$

How does look the radiation emitted by atom under these conditions?
Resonance polarization for normal Zeeman triplet

\[ J_l = 0 \rightarrow J_u = 1 \]

\[ \frac{Q}{I} = -\frac{1 - \mu^2}{1 + \mu^2} \]

\[ I = \frac{3}{4} (1 + \cos^2 \mu) \]
Shape of atomic orbital (level with $j=1$) of mixed state

Relative weight of pure states $|\alpha jM\rangle$ can be quantified by means of density matrix.

$$w(M=-1) = w(M=+1) \times w(M=0)$$
Anisotropic velocity distribution of colliders may cause atomic **polarization** as well as anisotropic irradiation. Thermal distribution leads to level **depolarization**.

**The main problem in practice:**

Cross-section data for transitions $\alpha jM \rightarrow \alpha' j'M'$

Methods of solution: Semiclassical perturbation theory, close-coupling semiclassical calculations, Hartree-Fock equations,…
The effect of inelastic collisions with charged particles

Example: 3-principal level hydrogen without fine structure splitting

The main depolarizing effect is connected to transitions $nl \rightarrow nl'$ induced by thermal electrons and protons.
Local atomic equilibrium

Atomic density matrix can be calculated from known physical properties (we limit our analysis to stationary problems):

• Stokes parameters \((I,Q,U,V)^T\)

• Velocity distributions of perturbers (thermal and nonthermal)

using the equations of statistical equilibrium (ESE):

\[ \Pi \rho = n \]

The matrix \(\Pi\) contains radiative and collisional rates for individual transitions between Zeeman sublevels \(a_jM\).

(Sahal-Bréchot, 1977), (Bommier, 1980)
The radiative transfer equation for polarized radiation can be derived by the similar procedure as ESE: By the perturbative solution of Liouville equation and calculation of evolution of operators corresponding to Stokes parameters.

The solution reads:

$$\frac{dS}{dz} = \varepsilon - \eta S$$

with $S$ being the Stokes vector, $z$ parametrization of ray path, $\varepsilon$ is 4-component emission vector, and $\eta$ is the 4-by-4 matrix of absorption (+ stimulated emission).

The radiative transfer equation (RTE) is formally same as in unpolarized case. The main difference is coupling of all the Stokes parameters so that even scalar intensity $I$ is affected by polarization state.

(Landi Degl’Innocenti, 1984)
NLTE self-consistency loop: Nonlinear radiation transfer solution

1. **Find density matrix elements**
2. **RTE for Stokes parameters**
3. **Find Stokes parameters for each direction of radiation field**
4. **ESE for density matrix**

The process involves:
- Writing Stokes parameters
- Solving for Stokes parameters
- Writing density matrix elements
- Solving for density matrix
Solution of radiation transfer I.
Operator splitting for polarized radiation

1. The mean radiation tensor $J$ has to be calculated from Stokes parameters (integration over line profile and directions, …)

2. Radiation field is formally expressed as a function of density matrix $\rho$ along the atmosphere via $\Lambda$-operator matrix.

3. We extract the diagonal of specific intensity component of $\Lambda$-operator and use linearization of statistical equilibrium equations in level populations.

4. The formal solver of RTE is based on short characteristics method.

5. The convergence rate of this Jacobi (ALI) scheme is $O(N^2)$ (where $N$ is number of spatial discretization points). (Manso Sainz & Trujillo Bueno, 2003)

There are too many states we have to take into account. Therefore the solution is slow. We should use more effective integration methods. The choice can be a multigrid technique.
The basic idea of multigrid technique is the following:

Iterative methods (such as the Jacobi method) tend to damp out high frequency components of the error fast, but converges poorly due to slow reduction of low-frequency components. The idea is to use the set of differently coarsed grids to reduce both the short- and long-period errors. The ALI iteration shall be used as a smoothing procedure of the error in the individual grids.

The era of extensive development of MG methods started in 1970's by the work of Brand (1977) and it is used especially by numerical solvers of elliptic boundary value problems. Several steps in using of the MG methods for purposes of unpolarized radiative transfer were made by Steiner (1991), Väth (1994) and Fabiani Bendicho et al. (1997).
Solution of radiation transfer III.

Convergence rate of MG

Restriction and prolongation operators are used to transform data between grids.

Algorithm:

1. Smooth error in fine grid → the error is now smooth.
2. Project into coarse grid together with residuum of the solution.
3. Calculate solution at coarse grid an interpolate density matrix corrections to the fine grid.
4. Go to step 1.

1D, 5-grids, finest grid: 257 points:

The convergence rate is $O(N)$. 

Graph showing convergence over CPU time.
The standard simplest model:
2-level atom, isothermal semi-infinite atmosphere, constant photon destruction probability $\epsilon$, no collisional depolarization
Resonance polarization degree is of few percent at limb and has **tangential orientation** in line centre.
Radial v. tangential polarization

Radial orientation

Tangential orientation
Impact polarization in Solar flares?

The magnetic energy is being released by accelerated charged particles. The question is: If there are proton beams propagating into the chromosphere, is it possible to detect them due to impact polarization of hydrogen levels?
Evidence of polarization

There is some linear polarization:

There is no linear polarization:

There are observations since 1990’s which indicate linear polarization of the order of >5% in flares. The orientation is preferentially radial, sometimes tangential.

Other measurements indicate no linear polarization in wide range of flares measurements.

(Xu et al., 2005), ...

(Bianda et al., 2005)
Proton beams in flaring chromosphere

Energy distribution of the beam:

*Initial distribution:*

\[ F(E) \sim E^{-\delta}, \quad E > E_0 \]

The initial power-law distribution changes its shape as the beam is decelerated in collisions with ambient electrons and hydrogen.

Input chromosphere models: semiempirical models F1, VAL F,...
(Machado et al., 1980), (Vernazza et al., 1981)

Electron volume density for initial VAL F model:

Fluxes: \(0, 10^8, 10^9, 10^{10}, 10^{11}\) ergs cm\(^{-2}\) s\(^{-1}\).

The proton beam is propagating through the chromosphere and ionizing the hydrogen atoms.
How is local Hα impact polarization?

Degree of polarization is defined as $Q/I$

Assume that Hα is be **optically thin** line in upper chromospheric layers (Vogt et al., 1997, 2001). The orientation of emitted polarization is **radial**. and most significant polarization is induced by **4-5 keV** protons.

(Vogt et al., 2001) (Štěpán et al., 2006)

The **higher density** of background electrons and protons leads to much higher depolarization effect.
Selfconsistent NLTE solution of radiation field leads to modified theoretical line profile and fractional polarization $Q/I$.

The effect of impact polarization is most significant in VAL F model where total tangential resonance polarization is reduced by impacts. The total polarization degree cannot exceed few tenths of %. More suitable hot models (F1) show completely negligible Hα polarization (~0.02%): much lower than it is observed.
What’s the source of observed polarization?

Candidates: impact polarization by electron beams and return currents induced by these beams. (Hénoux & Karlický, 2003)