The Solution of the Radiative Transfer Equation in Accretion Discs of Cataclysmic Variables

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Abstract. A method for solving the radiative transfer equation appropriate for the case of a white dwarf with an accretion disc is described. The problem is solved in two dimensions in axial symmetry. The velocity field in the radiative transfer equation is not taken into account using the Sobolev approximation, but rather in detail using the Lorentz transformation. This allows us to include both small and high velocity gradients present in accretion discs.

1. Introduction

Cataclysmic variables are systems, where one of the components is a white dwarf that is accreting mass from a companion. Since light from these systems is dominated by emission from the disc both in outburst and during the quiescence phase, the disc cannot be neglected in the calculation of synthetic spectra. We present here a method, which is able to include not only the radiation of both white dwarf and disc, but also of the hot corona and polar wind.

2. Description of the Model

Based on a specific system we present our method of solution of the radiative transfer equation in a CV system including the white dwarf, disc, and circumstellar region. First, we set a disc model, from which we obtain input parameters for a solution of the radiative transfer equation using our code (Korčáková & Kubát 2003, 2004).

2.1. Physical Model of the Disc

We choose the cataclysmic variable HT Cas as a source of input parameters for our tests. The parameters of this system were taken from Horne et al. (1991).

\[
\begin{align*}
M_{wd} &= 0.6 M_{\odot} \\
R_{wd} &= 8.3 \cdot 10^8 \text{ cm} \\
T_{\text{eff,wd}} &= 12000 K \\
M_e/M_{wd} &= 0.15 \\
a &= 0.66 R_{\odot} \\
R_{\text{out(disc)}} &= 0.23 a
\end{align*}
\]

Here \(a\) is the distance between the center of white dwarf and the L1 point. We choose the electron concentration in the corona to be \(n_e = 10^{13} \text{ cm}^{-3}\). The electron concentration in the mid-plane of the disc was adopted from Williams (1980)

\[
n_e(r) = \frac{1}{4\pi \beta} \left( \frac{\mu}{km_H} \right)^{1/2} \left( \frac{8\pi \sigma \xi(\tau_e)}{3GM_{wd}} \right)^{1/8} \dot{M}^{7/8} r^{-13/8},
\]

(1)
where $\sigma$ is the Stefan-Boltzmann constant, $\mu$ is the mean molecular weight, and $m_H$ is the mass of hydrogen. $1/\beta = Re$ is the Reynolds number. For our case we choose $Re = 5000$, the parameter $\xi = 2/3$, and a mass transfer rate of $\dot{M} = 2 \times 10^{-9} M_\odot$/year. The decrease of $n_e$ with distance from the equator is described by the equation (Williams & Shipman 1988)

$$n_e(z) = n_e(z_0)e^{-(z/H)^2/2}, \quad \text{where} \quad H = R_{\text{out}} \left( \frac{R_{\text{out}} kT}{GM_{\text{wd}} \mu m_p} \right)^{1/2}, \quad (2)$$

and $m_p$ is the proton mass. The temperature dependence on radial distance in the mid-plane of the disc is

$$T_{\text{eff}}(r) = T_* x^{-3/4}(1 - x^{-1/2})^{1/4}, \quad \text{where} \quad T_* = \left( \frac{3GM_{\text{wd}} \dot{M}}{8\pi R_{\text{wd}}^3 \sigma} \right)^{1/4} \quad (3)$$

and $x = r/R_{\text{wd}}$. We consider no temperature changes with the angle $\theta$. For the velocity field we adopt the Kepler velocity law $v = (gM_{\text{wd}}/r)^{1/2}$.

2.2. Solution of the Radiative Transfer Equation

We assume axial symmetry in our model and choose a spherical grid. Our method is derived from the general method described in Korčáková & Kubát (2004), however, the grid is chosen to be substantially finer near the equatorial plane (see Fig. 1) to obtain a more accurate geometric description of the disc geometry. To reduce a 3D problem to a 2D one we solve the radiation transfer equation in a set of longitudinal planes. The whole radiation field is then obtained by rotating the planes around the axis of symmetry.

In every plane the transfer problem is solved using a combination of the short and long characteristics methods (Fig. 2). The ray starts and ends at the
grid circle, so it’s possible to intersect more cells. This allows us to include the global character of the radiative field better. The transfer equation is then solved along the selected rays

\[ I(B) = I(A) e^{-\Delta \tau_{AB}} + \int_0^{\Delta \tau_{AB}} S(t) e^{-\int_0^t (\Delta \tau_{AB})} dt. \]  

The interval \( AB \) is a section of the ray within each cell. We assume the source function and the opacity to change linearly within this interval.

In every cell we assume a constant velocity and we only allow a change of velocity at the cell boundaries. This approximation allows us to solve the static equation of the radiative transfer in the cells. At the cell boundaries we perform the transformation of frequency (we can neglect the transformation of intensity).

3. Preliminary Results

Non-rotating disc  First, let us have a look at the limiting case of a nonrotating disc. Since the disc is geometrically very thin and the physical quantities change of the order of magnitude in the \( r \) scale as well as in the distance from equator, there is a large difference in the line profile depending on the angle of sight (disc inclination with respect to the observer). In Fig. 3 (left panel) we plot H\( \alpha \) line profiles for different inclination angles (\( i = 0^\circ, 30^\circ, 60^\circ, 90^\circ \)).

Rotating disc  In Figure 3 (right panel), we show an H\( \alpha \) line profile for a rotational velocity of \( v(r) = 0.1v_{\text{Kepler}}(r) \). For a more realistic velocity field in the disc (faster rotation) the grid of longitudinal planes as well as the frequency grid must be much finer to resolve all the velocity gradients well and, consequently,
the computing time is substantially longer. Here we presented an illustrating case of a disc with somewhat slower rotation, which is sufficient to show the usefulness of the method. We are working on accelerating the code.

4. Conclusion

We present a method for solving the radiative transfer equation in axial symmetry with a velocity field. This method takes an advantage of the disc geometry, which allows us to include the changes of the physical properties in the disc with the radial distance and the distance from the equatorial plane. Another advantage of this method is the solution of the radiative transfer equation in the white dwarf and in the disc as one system. It means that the disc radiation, radiation from the central object as well as from the boundary region, wind and corona are naturally and simultaneously included in the calculations. With our method it is possible to solve the radiative transfer equation both for optically thin and thick discs.

Acknowledgments. This research has made use of NASA’s Astrophysics Data System. This work was supported by grants GA ČR 205/04/1267 and 205/04/P224. The Astronomical Institute Ondřejov is supported by projects K2043105 and Z1003909.

References