

WINDS OF HOT MASSIVE STARS

II Lecture: Basic theory of winds of hot massive stars

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Selected Topics in Astrophysics

Faculty of Mathematics and Physics

October 16, 2013

Prague

- 1 Properties of winds of hot massive stars
- 2 Line-driven wind theory
- 3 Wind hydrodynamic equations
- 4 Radiative force
- 5 Sobolev approximation

Recommended literature

- 1 *Lamers, H. G. L. M.; Cassinelli, J. P.: Introduction to Stellar Winds*
Cambridge University Press, 1999
- 2 *Fullerton, A. W.: Observations of Hot-Star Winds*
Lecture Notes in Physics, Vol. 497, pp. 187-237, 1997
- 3 *Kudritzki, R.-P.; Puls, J.: Winds from Hot Stars*
Annual Review of Astronomy and Astrophysics, Vol. 38, pp. 613-666, 2000
- 4 *Puls, J.; Vink, J. S.; Najarro, F.: Mass loss from hot massive stars*
The Astronomy and Astrophysics Review, Vol. 16, Issue 3-4, pp. 209-325, 2008
- 5 *Krtićka, J.; Kubát, J.: Radiatively Driven Winds of OB Stars - from Micro to Macro* ASP Conference Series, Vol. 361, p. 153, 2007

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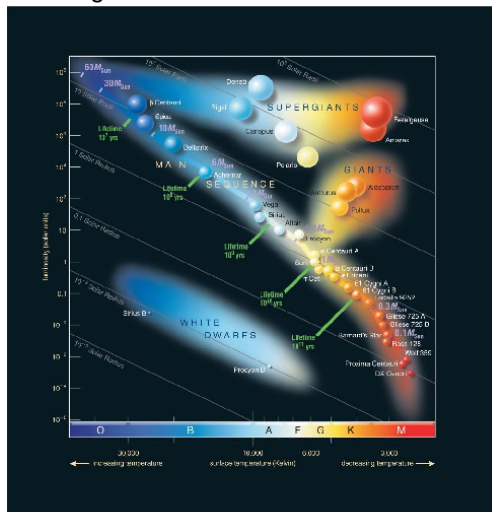
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Properties of winds of hot massive stars

- **EXTREMELY LUMINOUS**
spectral types A, B, and O;
 $L \gtrsim 10^2 [L_{\odot}]$
W-R, LBV, B[e] stars
- **HOT** - $T_{\text{eff}} \gtrsim 8000 \text{ [K]}$
- **MASSIVE** - $M \gtrsim 2 [M_{\odot}]$

H-R diagram



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Typical parameters for O-type stars

| Parameter | Sun | O-type stars |
|---------------------------------------|------------|--------------------|
| $L [L_{\odot}]$ | 1 | $\sim 10^6$ |
| $T_{\text{eff}} [\text{K}]$ | 6000 | $\gtrsim 30\,000$ |
| $M [M_{\odot}]$ | 1 | $\gtrsim 8$ |
| total life time [yr] | 10^{10} | $\sim 10^7$ |
| $T_{\text{wind}} [\text{K}]$ | 10^6 | $\sim 10^4$ |
| $\dot{M} [M_{\odot} \text{ yr}^{-1}]$ | 10^{-14} | $\sim 10^{-6}$ |
| $v_{\infty} [\text{km s}^{-1}]$ | 400 (700) | $\sim 10^2 - 10^3$ |

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- **SHORT LIFETIMES**
($\sim 10^6 \text{ yr}$)
- **END IN SUPERNOVA EXPLOSION**

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($\sim 10^6$ yr)

- **END IN SUPERNOVA EXPLOSION**

- **TYPICAL \dot{M}**

from 10^{-7} to $10^{-4} M_{\odot}$

- **TYPICAL v_{∞}** - from 200 km s^{-1} (for A-supergiant) to 3000 km s^{-1} (for early O-stars)

Typical parameters for O-type stars

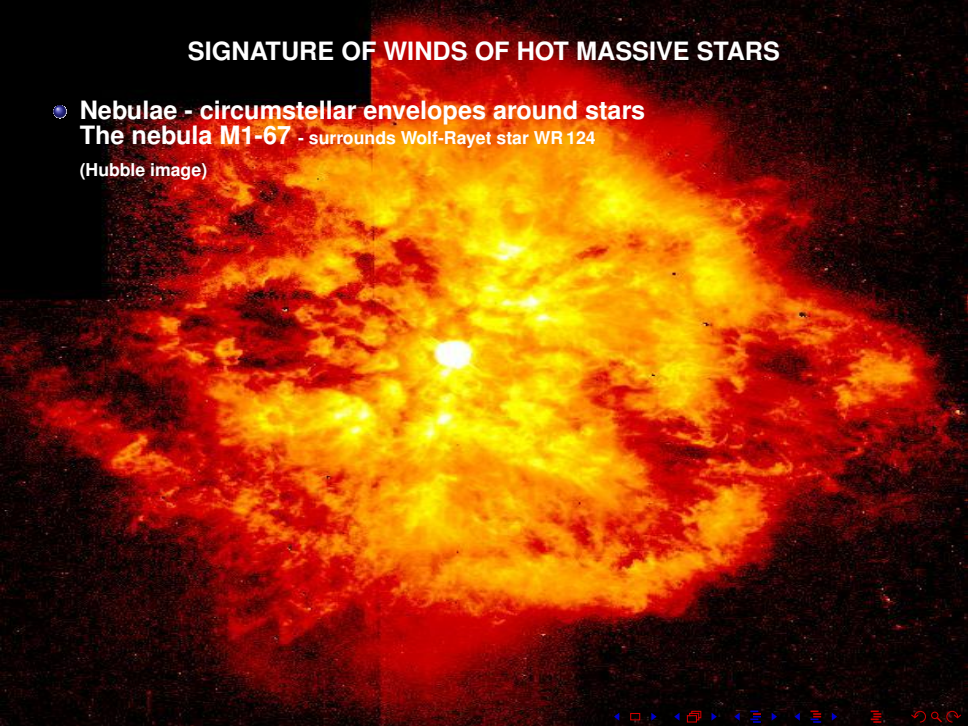
| Parameter | Sun | O-type stars |
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| $L [L_{\odot}]$ | 1 | $\sim 10^6$ |
| $T_{\text{eff}} [\text{K}]$ | 6000 | $\gtrsim 30000$ |
| $M [M_{\odot}]$ | 1 | $\gtrsim 8$ |
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SIGNATURE OF WINDS OF HOT MASSIVE STARS

- **Nebulae - circumstellar envelopes around stars**

The nebula M1-67 - surrounds Wolf-Rayet star WR 124

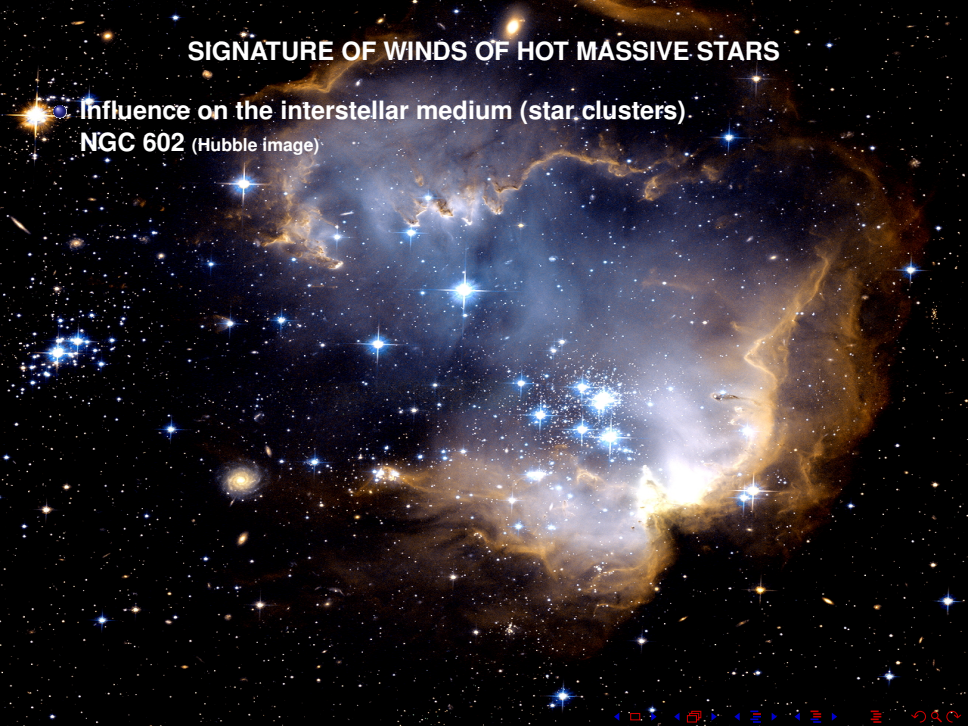
(Hubble image)



SIGNATURE OF WINDS OF HOT MASSIVE STARS

● Influence on the interstellar medium (star clusters)

NGC 602 (Hubble image)



Properties of winds of hot massive stars

- Hot stars emit their peak radiation in the UV wavelength region

$$\lambda_{\max} T = b$$

$$b = 0.29 \text{ cm K}; \quad T = 30\,000 \text{ K} \Rightarrow \lambda_{\max} = 960 \text{ \AA}$$

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- The outer atmospheres of hot stars have plenty of absorption lines in the ultraviolet (e.g., resonance lines from N V $\lambda\lambda$ 1239, 1243 Å, Si IV $\lambda\lambda$ 1394, 1403 Å, C IV $\lambda\lambda$ 1548, 1551 Å, Morton 1967)

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- Massive hot stars are luminous \Rightarrow accelerating force: **RADIATIVE FORCE**
- Winds can be driven by **absorption of radiation in spectral lines** (Lucy & Solomon, 1970)

Line-driven wind theory

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- Modern studies of hot stars' winds were stimulated mainly by UV observations
- Pioneering works of Lucy & Solomon (1970) and Castor, Abbott, & Klein (1975, CAK) serve as a basis for present hot star wind theory

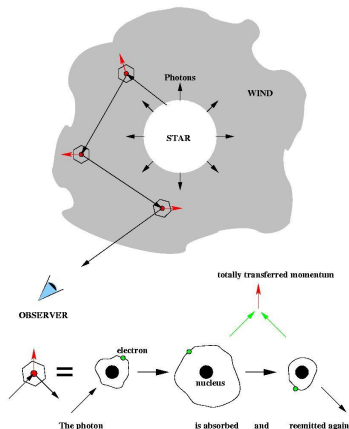
Principle of radiative line-driving

Hot star winds are accelerated via a two-step process:

- 1 The photons are scattered in lines of ions of heavier elements (e.g., C, N, O, Ne, Si, P, S, Ni, Fe-group elements etc.)
 - physical process: **momentum and energy transfer** by absorption and scattering
- 2 The outward accelerated ions transfer their momenta to the bulk plasma of the wind (hydrogen and helium - mostly passive component)
 - physical process: **Coulomb collisions**

Principle of radiative line-driving

The principle of radiatively driven winds



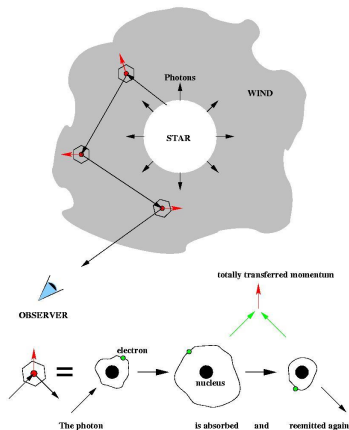
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1 The light scattering in lines of heavier elements

- Photons transfer (part of) their *momentum* to heavier ions and electrons by **line scattering**
 - photon is absorbed by an ion

Principle of radiative line-driving

The principle of radiatively driven winds



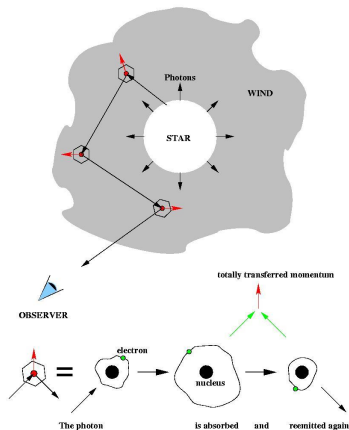
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Principle of radiative line-driving

The principle of radiatively driven winds



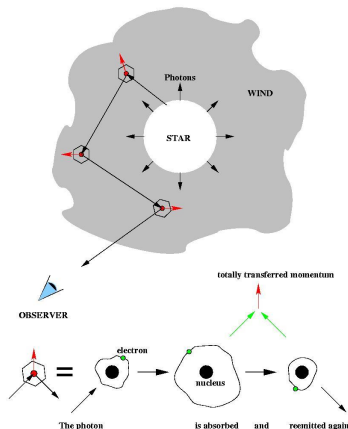
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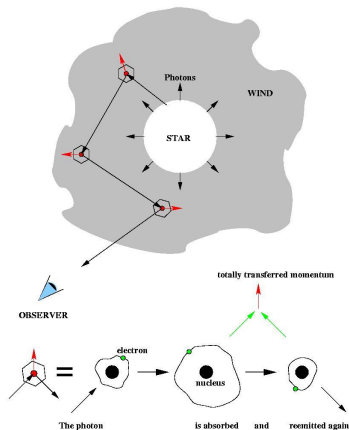
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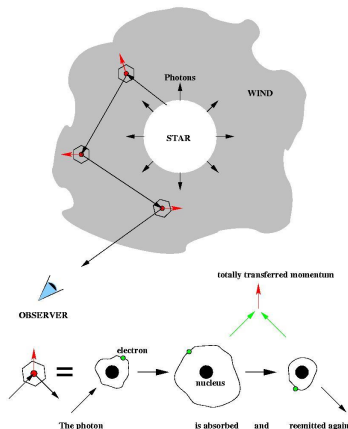
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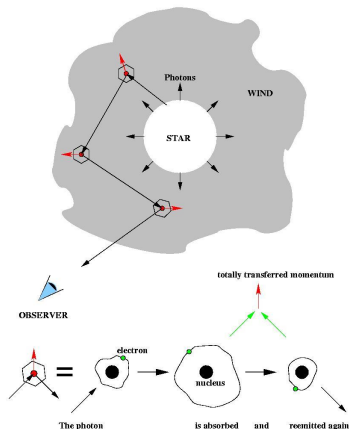
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Principle of radiative line-driving

The principle of radiatively driven winds



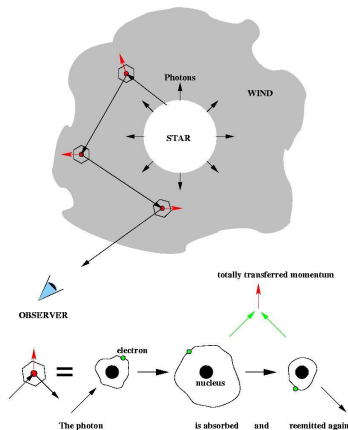
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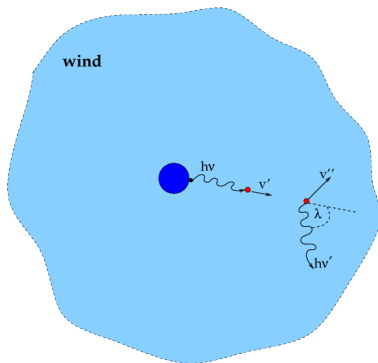


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 - momentum is transferred to the ion
 - electron "falls" back to its ground state or to a different, low-energy orbit
 - a "new" photon is emitted
 - the ion is accelerated into the opposite direction of the photon
 - resulting net-acceleration of the ion due to absorption and emission is the vector-sum of both accelerations
 - only the outward directed acceleration due to absorption processes survives

Principle of radiative line-driving



1 The light scattering in lines of heavier elements

- momentum of an ion after absorption of photon

$$mv'_r = mv_r + \frac{hv}{c}$$

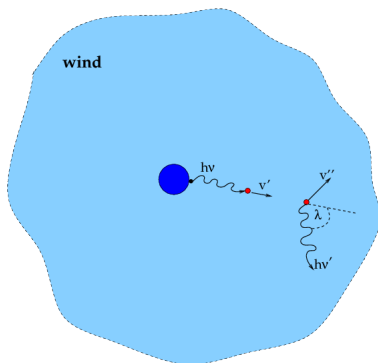
- increase of velocity

$$\Delta v_r = \frac{hv}{c}$$

- momentum of an ion after emission of photon

$$mv''_r = mv'_r - \frac{hv'}{c} \cos \lambda$$

Principle of radiative line-driving



1 The light scattering in lines of heavier elements

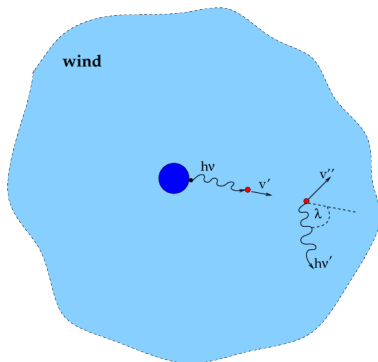
- frequency of absorbed photon in observer frame

$$\nu = \nu_0 \left(1 + \frac{v_r}{c}\right)$$

- frequency of emitted photon in observer frame

$$\nu' = \nu_0 \left(1 + \frac{v_r'}{c}\right)$$

Principle of radiative line-driving



1 The light scattering in lines of heavier elements

- velocity of the ion after absorption and re-emission

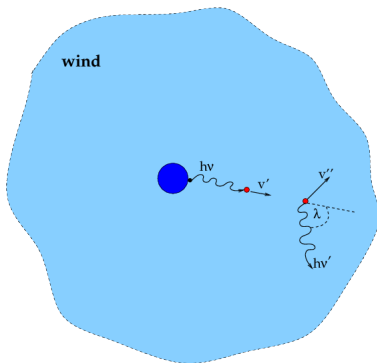
$$v_r'' = v_r + \frac{h\nu_0}{mc} \left(1 + \frac{v_r}{c}\right) - \frac{h\nu_0}{mc} \left(1 + \frac{v_r'}{c}\right) \cos \lambda$$

- for $v \ll c$ and $h\nu_0 \ll c$

$$\Delta v_r = v_r'' - v_r = \frac{h\nu_0}{mc} (1 - \cos \lambda)$$

- **forward scattering** ($\cos \lambda = 1$) \Rightarrow the momentum does not increase
- **backward scattering** ($\cos \lambda = -1$) \Rightarrow the momentum increases by $2h\nu_0/c$
- re-emission of photons is in random direction

Principle of radiative line-driving



1 The light scattering in lines of heavier elements

- velocity of the ion after absorption and re-emission

$$v_r'' = v_r + \frac{hv_0}{mc} \left(1 + \frac{v_r}{c}\right) - \frac{hv_0}{mc} \left(1 + \frac{v_r'}{c}\right) \cos \lambda$$

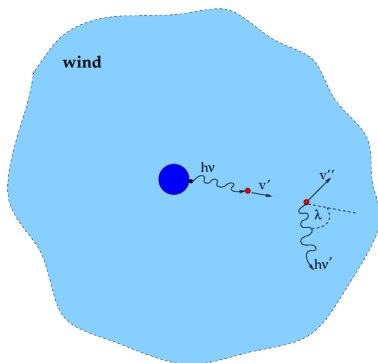
- for $v \ll c$ and $hv_0 \ll c$

$$\Delta v_r = v_r'' - v_r = \frac{hv_0}{mc} (1 - \cos \lambda)$$

- the mean transfer of momentum

$$\langle m\Delta v \rangle = \frac{hv_0}{c} \frac{1}{4\pi} \int_{-\pi/2}^{\pi/2} (1 - \cos \lambda) 2\pi \sin \lambda \, d\lambda = \frac{hv_0}{c}$$

Principle of radiative line-driving

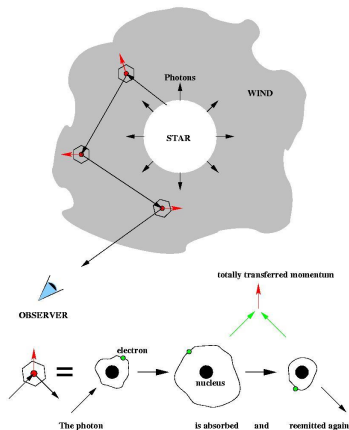


1 The light scattering in lines of heavier elements

- line scatterings are of bound-bound type, i.e., line transitions
- the wind acceleration is due to **RADIATIVE LINE DRIVING**

Principle of radiative line-driving

The principle of radiatively driven winds

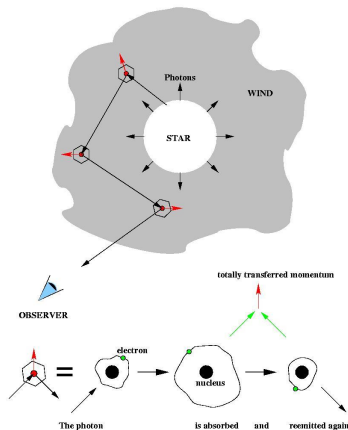


from homepage of Joachim Puls

- 1 The light scattering in lines of heavier elements
- 2 Momentum transfer by Coulomb coupling
 - the outward accelerated ions transfer their momenta to the bulk plasma of the wind (basically H and He) via Coulomb collisions

Principle of radiative line-driving

The principle of radiatively driven winds

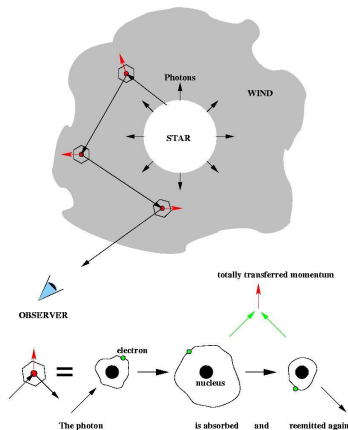


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- 1 The light scattering in lines of heavier elements
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 - the outward accelerated ions transfer their momenta to the bulk plasma of the wind (basically H and He) via Coulomb collisions
 - the total wind is accelerated outward

Principle of radiative line-driving

The principle of radiatively driven winds



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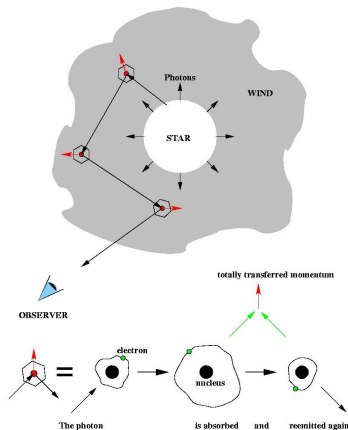
- 1 The light scattering in lines of heavier elements
- 2 Momentum transfer by Coulomb coupling
 - Condition for the Coulomb coupling to be efficient

$$t_s < t_d$$

- t_s [s] - characteristic time for slowing down heavier ions by collisions
- t_d [s] - time takes the heavier ions to gain a large drift velocity with respect to H and He
- first shown by Lucy and Solomon (1970) and improved by Lamers and Morton (1976)

Principle of radiative line-driving

The principle of radiatively driven winds



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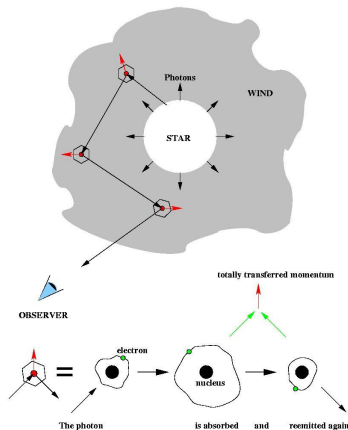
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 - Condition for the Coulomb coupling to be efficient

$$t_s = 0.305 \frac{A}{Z^2} \frac{T_e^{3/2}}{n_e(1 - 0.022 \ln n_e)}$$

- A - mass of charged particles (in units of m_H)
- Z - charge (in units of the electron charge) due to interaction with H^+ , He^{++} and electrons
- n_e - the electron density
- for winds with $10^8 \leq n_e \leq 10^{12} \Rightarrow (1 - 0.022 \ln n_e) \approx 0.5$

Principle of radiative line-driving

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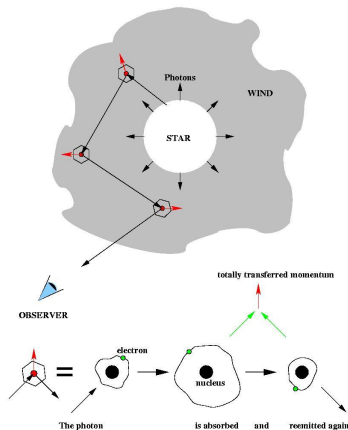
$$t_d = \frac{v_{th}}{g_i}$$

$$v_{th} = \sqrt{\frac{2k_B T_e}{m_H A_f}}$$

- A_f - atomic mass for field particles ($A_f \approx 1$ for protons)
- g_i - acceleration of the absorbing ions
- T_e - temperature of the wind

Principle of radiative line-driving

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 - Momentum transfer from photons to ions

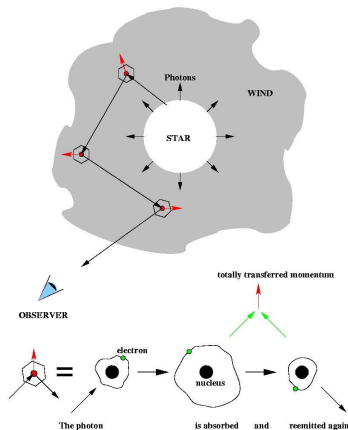
$$\frac{d(mv)}{dt} = Am_H g_i = \frac{\pi e^2}{m_e} f \frac{\mathcal{F}_{\nu_0}}{c}$$

$$\mathcal{F}_{\nu_0} = \mathcal{F}_{\nu_0}^* \left(\frac{R_*}{r} \right)^2$$

- $(\pi e^2 / m_e c) f$ - cross section for absorption
- \mathcal{F}_{ν_0} - flux at distance r from the star at the frequency of the line ν_0
- $\mathcal{F}_{\nu_0}^* = L_{\nu_0}^* / 4\pi R_*^2$ - flux at surface of the star

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- 2 Momentum transfer by Coulomb coupling
 - Condition for efficient Coulomb coupling

$$\frac{L_{\nu_0}^* T_e}{4\pi r^2 n_e} < \frac{Z^2 c}{0.61} \sqrt{2k_B m_H} \left(\frac{\pi e^2}{m_e c} f \right)^{-1} A_f^{-1/2} = 3.6 \times 10^{-6} \quad (1)$$

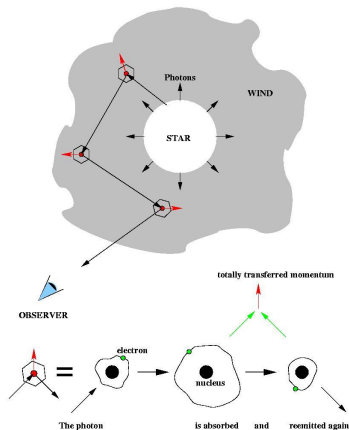
- $A_f = 1$, $f = 0.1$, and $Z = 3$
- $T_e \approx 0.5 T_{\text{eff}}$, $L_{\nu_0}^* T_e = 5.26 \times 10^{-12} L_*$;
 $n_e = 5.2 \times 10^{23} \text{ g cm}^{-3}$

$$\frac{L_* v}{\dot{M}} < 5.9 \times 10^{16}$$

- for hot stars this is satisfied

Principle of radiative line-driving

The principle of radiatively driven winds

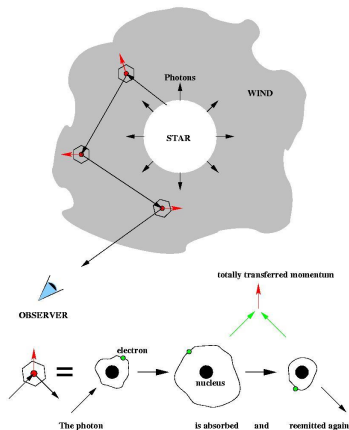


from homepage of Joachim Puls

- 1 The light scattering in lines of heavier elements
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 - hydrogen and helium are mostly passive components of the wind (inefficient for wind driving)

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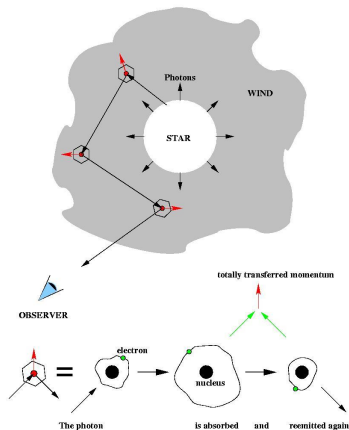


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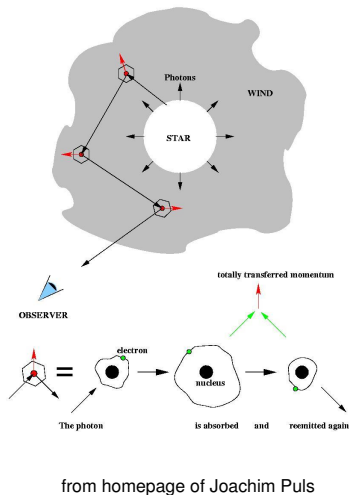


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 - if transfer of momentum between metallic and passive wind component is efficient, the wind is well-coupled and can be treated as one component (Castor et al., 1976)

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The principle of radiatively driven winds



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 - hydrogen and helium are mostly passive components of the wind (inefficient for wind driving)
 - metal lines are responsible for the line driving
 - if transfer of momentum between metallic and passive wind component is efficient, the wind is well-coupled and can be treated as one component (Castor et al., 1976)
 - if the transfer of momentum is inefficient, the wind components may decouple (Springmann and Pauldrach, 1992, Krtićka and Kubát 2000)

Wind hydrodynamic equations

Single-fluid treatment, neglecting viscosity and forces due to electric and magnetic fields

- equations of the continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

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- equations of motion (momentum)

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \mathbf{g}_{\text{ex}}$$

- $\mathbf{v} = \mathbf{v}(\mathbf{r}, t)$ - velocity field
- $\rho = \rho(\mathbf{r}, t)$ - mass density
- $p = p(\mathbf{r}, t)$ - gas pressure
- \mathbf{g}_{ex} - external acceleration; $\mathbf{g}_{\text{ex}} = \mathbf{g}_{\text{grav}} + \mathbf{g}_{\text{rad}}$

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- \mathbf{g}_{ex} - external acceleration; $\mathbf{g}_{\text{ex}} = \mathbf{g}_{\text{grav}} + \mathbf{g}_{\text{rad}}$
- energy equation
 - an approximate solution of the energy equation is allowed (see Klein and Castor, 1978)
 - T_e is approximately constant with radius and slightly less than T_{eff} , i.e.

isothermal wind

Wind hydrodynamic equations

Assumption: stationary and spherically symmetric wind

- equations of the continuity

$$\frac{1}{r^2} \frac{d}{dr} (\rho v_r r^2) = 0$$

- after integration \Rightarrow total outward mass flux, i.e. \dot{M}

$$\dot{M} \equiv \frac{dM_*}{dt} = 4\pi \rho(r) v_r(r) r^2 = \text{const.}$$

Wind hydrodynamic equations

Assumption: stationary and spherically symmetric wind

- equations of motion (momentum)

$$v_r \frac{dv_r}{dr} = -\frac{1}{\rho} \frac{dp}{dr} - \mathbf{g}_{\text{grav}} + \mathbf{g}_{\text{rad}}$$

- $g_{\text{grav}} = GM_*/r^2$ (G - the gravitational constant)
- the gas pressure p is given by an ideal gas equation of state

$$p = \frac{\rho k_B T}{\mu m_H} = \rho a^2$$

- a - isothermal speed of sound (const.)
- k_B - Boltzmann's constant
- m_H - the mass of a hydrogen atom
- μ - the mean molecular weight of gas particles

Wind hydrodynamic equations

Assumption: stationary and spherically symmetric wind

- equations of motion (momentum)

$$v_r \frac{dv_r}{dr} = -\frac{a^2}{\rho} \frac{d\rho}{dr} - \frac{GM_*}{r^2} + g_{\text{rad}}$$

- $g_{\text{grav}} = GM_*/r^2$ (G - the gravitational constant)
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Wind hydrodynamic equations

Assumption: stationary and spherically symmetric wind

- equations of motion (momentum)

$$\rho v_r \frac{dv_r}{dr} = -a^2 \frac{d\rho}{dr} - \frac{\rho G M_*}{r^2} + f_{\text{rad}}$$

- $f_{\text{grav}} = \rho G M_* / r^2$ - gravitational force
- f_{rad} - radiative force
- the gas pressure p is given by an ideal gas equation of state
- a - isothermal speed of sound (const.)

Radiative force

- f_{rad} - force due to a radiation field at a point r

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} d\nu \oint_{\Omega=4\pi} (\chi(r, \nu) I(r, \nu, \mathbf{k}) - \eta(r, \nu)) \mathbf{k} d\Omega$$

- χ_{ν} - absorption coefficient
- η_{ν} - emission coefficient
- I_{ν} - radiative intensity
- \mathbf{k} - unit vector of the direction of the radiation propagation
- For isotropic emissivity, the integral over all angles vanishes as well as the second term, and $\chi(r, \nu)$ can be factored out of angular integration

Radiative force

- f_{rad} - force due to a radiation field at a point r

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) d\nu \oint_{\Omega=4\pi} I(r, \nu, \mathbf{k}) \mathbf{k} d\Omega = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- χ_{ν} - absorption coefficient
- I_{ν} - radiative intensity
- \mathbf{k} - unit vector of the direction of the radiation propagation
- \mathcal{F} - radiation flux

$$\mathcal{F}(r, \nu) = \oint_{\Omega=4\pi} I(r, \nu, \mathbf{k}) \mathbf{k} d\Omega$$

Radiative force

- Total radiative force

$$f_{\text{rad}}(r) = f_{\text{cont}}(r) + f_{\text{line}}^{\text{tot}}(r)$$

- $f_{\text{cont}}(r)$ - force due to continuum opacity
- $f_{\text{line}}^{\text{tot}}(r)$ - force due to an ensemble of spectral lines

- Continuum opacity

- continuum processes: atomic free-free and bound-free transitions and scattering on free electrons
- continuum opacity due to free-free and bound-free processes can be neglected in the winds of O and B type stars
- scattering of free electrons (Thomson scattering) - the main contributor to the continuum opacity

Radiative force

- Radiative force

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) d\nu \oint_{\Omega=4\pi} I(r, \nu, \mathbf{k}) \mathbf{k} d\Omega = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- Radiative force due to radiation scattering on free electrons

$$f_{\text{cont}}(r) = \frac{\chi_{\text{th}}(r)}{c} \int_{\nu=0}^{\infty} d\nu \oint_{\Omega=4\pi} I^{\text{c}}(r, \nu, \mathbf{k}) \mathbf{k} d\Omega = \frac{\chi_{\text{th}}(r)}{c} \int_0^{\infty} \mathcal{F}(r, \nu) = \frac{n_{\text{e}}(r) \sigma_{\text{Th}} L}{4\pi r^2 c}$$

- I^{c} is "direct" continuum intensity from the stellar surface
- χ_{th} - the Thomson scattering opacity

$$\chi_{\text{th}}(r) = n_{\text{e}}(r) \sigma_{\text{Th}}$$

- $\sigma_{\text{Th}} = 6.65 \times 10^{-25} \text{ cm}^2$ - the cross-section for Thomson scattering
- n_{e} - the number density of free electrons

- $L = 4\pi r^2 \int_0^{\infty} \mathcal{F}(r, \nu)$

Radiative force

- Radiative force

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) d\nu \oint_{\Omega=4\pi} I(r, \nu, \mathbf{k}) \mathbf{k} d\Omega = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- Radiative force due to radiation scattering on free electrons

$$f_{\text{cont}}(r) = \frac{\chi^{\text{th}}(r)}{c} \int_{\nu=0}^{\infty} d\nu \oint_{\Omega=4\pi} I^c(r, \nu, \mathbf{k}) \mathbf{k} d\Omega = \frac{\chi^{\text{th}}(r)}{c} \int_0^{\infty} \mathcal{F}(r, \nu) = \frac{n_e(r) \sigma_{\text{Th}} L}{4\pi r^2 c}$$

- Ratio between the force due to the light scattering on free electrons and the gravitational force - **Eddington factor** (luminosity-to-mass ratio)

$$\Gamma_e = \frac{f_{\text{cont}}}{f_{\text{grav}}} = \frac{\sigma_{\text{Th}} \frac{n_e}{\rho(r)} L}{4\pi c G M}$$

- $\Gamma_e \rightarrow 1$ - the Eddington limit

Radiative force

- Radiative force

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) d\nu \oint_{\Omega=4\pi} I(r, \nu, \mathbf{k}) \mathbf{k} d\Omega = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- Radiative force due to radiation scattering on free electrons

$$f_{\text{cont}}(r) = \frac{\chi^{\text{th}}(r)}{c} \int_{\nu=0}^{\infty} d\nu \oint_{\Omega=4\pi} I^{\text{c}}(r, \nu, \mathbf{k}) \mathbf{k} d\Omega = \frac{\chi^{\text{th}}(r)}{c} \int_0^{\infty} \mathcal{F}(r, \nu) = \frac{n_{\text{e}}(r) \sigma_{\text{Th}} L}{4\pi r^2 c}$$

- comparison with the gravity force

$$\Gamma_{\text{e}} = \frac{f_{\text{cont}}}{f_{\text{grav}}} = \frac{\sigma_{\text{Th}} \frac{n_{\text{e}}}{\rho(r)} L}{4\pi c G M}$$

$$\Gamma_{\text{e}} = 10^{-5} \left(\frac{L}{L_{\odot}} \right) \left(\frac{M}{M_{\odot}} \right)^{-1}$$

Radiative force

- Radiative force

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) d\nu \oint_{\Omega=4\pi} I(r, \nu, \mathbf{k}) \mathbf{k} d\Omega = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- Radiative force due to radiation scattering on free electrons

$$f_{\text{cont}}(r) = \frac{\chi^{\text{th}}(r)}{c} \int_{\nu=0}^{\infty} d\nu \oint_{\Omega=4\pi} I^{\text{c}}(r, \nu, \mathbf{k}) \mathbf{k} d\Omega = \frac{\chi^{\text{th}}(r)}{c} \int_0^{\infty} \mathcal{F}(r, \nu) = \frac{n_{\text{e}}(r) \sigma_{\text{Th}} L}{4\pi r^2 c}$$

- radiative force due to the light scattering on free electrons is important, but it never exceeds the gravity force

Radiative force

- Radiative force

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- Radiative force due to line transition

$$\chi(r, \nu) = \frac{\pi e^2}{m_e c} \sum_{\text{lines}} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right) \phi_{ij}(\nu)$$

- $\phi_{ij}(\nu)$ - line profile; $\int_0^{\infty} \phi_{ij}(\nu) d\nu = 1$
- f_{ij} - oscillator strength
- $n_i(r)$, $n_j(r)$ level occupation number
- g_i - statistical weight of the level

Radiative force

- Radiative force

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$$f_{\text{line}}(r) = \frac{\pi e^2}{m_e c^2} \sum_{\text{lines}} \int_0^{\infty} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right) \phi_{ij}(\nu) \mathcal{F}(r, \nu) d\nu$$

- lines influence on $\mathcal{F}(r, \nu)$
- assumption: $\mathcal{F}(r, \nu)$ constant for frequencies corresponding to a given line,
 $\nu \approx \nu_{i,j}$

Radiative force

- Radiative force

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- Radiative force due to line transition

- maximum force

$$f_{\text{line}}^{\text{max}}(r) = \frac{\pi e^2}{m_e c^2} \sum_{\text{lines}} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right) \mathcal{F}(r, \nu_{i,j})$$

- $\nu_{i,j}$ - the line center frequency
- neglect of $n_j(r) \ll n_i(r)$
- $L_{\nu_{i,j}} = 4\pi r^2 \mathcal{F}(r, \nu_{i,j})$

Radiative force

- Radiative force

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- Radiative force due to line transition

- maximum force: comparison with gravity

$$\frac{f_{\text{line}}^{\text{max}}(r)}{f_{\text{grav}}(r)} = \frac{L e^2}{4m_e \rho G M c^2} \sum_{\text{lines}} f_{ij} n_i(r) \frac{L_{\nu_{i,j}}}{L}$$

- $\nu_{i,j}$ - the line center frequency
- neglect of $n_j(r) \ll n_i(r)$
- $L_{\nu_{i,j}} = 4\pi r^2 \mathcal{F}(r, \nu_{i,j})$

Radiative force

- Radiative force

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- Radiative force due to line transition

- maximum force: comparison with gravity

$$\frac{f_{\text{line}}^{\text{max}}(r)}{f_{\text{grav}}(r)} = \Gamma \sum_{\text{lines}} \frac{\sigma_{ij}}{\sigma_{\text{Th}}} \frac{n_i(r)}{n_e(r)} \frac{\nu_{i,j} L_{\nu}(\nu_{i,j})}{L}$$

$$\sigma_{ij} = \frac{\pi e^2 f_{ij}}{\nu_{i,j} m_e c}$$

- hydrogen: mostly ionised in the stellar envelopes $\Rightarrow n_i(r)/n_e(r)$ very small \Rightarrow negligible contribution to radiative force
- neutral helium: $n_i(r)/n_e(r)$ very small \Rightarrow negligible contribution to radiative force
- ionised helium: very small contribution to the radiative force

Radiative force

- Radiative force

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- Radiative force due to line transition

- maximum force: comparison with gravity

$$\frac{f_{\text{line}}^{\text{max}}(r)}{f_{\text{grav}}(r)} = \Gamma \sum_{\text{lines}} \frac{\sigma_{ij}}{\sigma_{\text{Th}}} \frac{n_i(r)}{n_e(r)} \frac{v_{i,j} L_{\nu}(v_{i,j})}{L}$$

$$\sigma_{ij} = \frac{\pi e^2 f_{ij}}{v_{i,j} m_e c}$$

- heavier elements (Fe, C, N, O, . . .): large number of lines, $\sigma_{ij}/\sigma_{\text{Th}} \approx 10^7 \Rightarrow f_{\text{line}}^{\text{max}}/f_{\text{grav}}$ up to 10^3
- radiative force may be larger than gravity (for many O stars $f_{\text{line}}^{\text{max}}/f_{\text{grav}} \approx 2000$, Abbott 1982, Gayley 1995) \Rightarrow **stellar wind**

Radiative force

- Radiative force

$$f_{\text{rad}}(r) = \frac{1}{c} \int_{\nu=0}^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

- Radiative force due to line transition

$$\chi(r, \nu) = \frac{\pi e^2}{m_e c} \sum_{\text{lines}} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right) \phi_{ij}(\nu)$$

$$f_{\text{line}}(r) = \frac{\pi e^2}{m_e c^2} \sum_{\text{lines}} \int_0^{\infty} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right) \phi_{ij}(\nu) \mathcal{F}(r, \nu) d\nu$$

- the main problem: the line opacity (lines may be optically thick) \Rightarrow
- necessary to solve the radiative transfer equation

Sobolev approximation

- Sobolev (1947) developed approach for treating line scattering in a rapidly accelerating flow
- This approximation is valid only if the velocity gradient is sufficiently large
- Due to the Doppler shift, the geometrical size in which a line can absorb photons with the fixed frequency is so small that χ_L and ρ change very little
- The profile function can be approximated with a δ -function that is sharply peaked around the central line frequency
- “Sobolev length”

$$L_S \equiv \frac{v_{\text{th}}}{dv/dr} \ll H \equiv \frac{\rho}{d\rho/dr} \approx \frac{v}{dv/dr}$$

- H - a typical flow variation scale
- $\rho/(d\rho/dr)$ and $v/(dv/dr)$ - the density and velocity scale length
- simplification of the calculation of f_{line} possible

The radiative transfer equation

Assumptions: spherical symmetry, stationary (time-independent) flow

$$\mu \frac{\partial}{\partial r} I(r, \mu, \nu) + \frac{1 - \mu^2}{r} \frac{\partial}{\partial \mu} I(r, \mu, \nu) = \eta(r, \mu, \nu) - \chi(r, \mu, \nu) I(r, \mu, \nu)$$

- frame of static observer
- $\mu = \cos \theta$
- $I(r, \mu, \nu)$ - specific intensity
- $\chi(r, \mu, \nu)$ - absorption (extinction) coefficient
- $\eta(r, \mu, \nu)$ - emissivity (emission coefficient)
- problem: $\chi(r, \mu, \nu)$ and $\eta(r, \mu, \nu)$ depend on μ due to the Doppler effect
- solution: use comoving-frame (CMF)

CMF radiative transfer equation

Assumptions: spherical symmetry, stationary (time-independent) flow

$$\mu \frac{\partial}{\partial r} I(r, \mu, \nu) + \frac{1 - \mu^2}{r} \frac{\partial}{\partial \mu} I(r, \mu, \nu) - \frac{\nu v(r)}{c r} \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{dv(r)}{dr} \right) \frac{\partial}{\partial \nu} I(r, \mu, \nu) = \eta(r, \nu) - \chi(r, \nu) I(r, \mu, \nu)$$

- $\chi(r, \mu, \nu)$ and $\eta(r, \mu, \nu)$ do not depend on μ
- neglected aberration, advection (unimportant for $v \ll c$)

CMF radiative transfer equation

The Sobolev transfer equation (Castor 2004)

$$\cancel{\mu \frac{\partial}{\partial r} I(r, \mu, \nu) + \frac{1 - \mu^2}{r} \frac{\partial}{\partial \mu} I(r, \mu, \nu)} - \frac{\nu v(r)}{c r} \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{dv(r)}{dr} \right) \frac{\partial}{\partial \nu} I(r, \mu, \nu) = \eta(r, \nu) - \chi(r, \nu) I(r, \mu, \nu)$$

- possible when $\frac{\nu v(r)}{c r} \frac{\partial}{\partial \nu} I(r, \mu, \nu) \gg \frac{\partial}{\partial r} I(r, \mu, \nu)$

CMF radiative transfer equation

Solution of the transfer equation for one line

$$-\frac{\nu v(r)}{c r} \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{dv(r)}{dr} \right) \frac{\partial}{\partial \nu} I(r, \mu, \nu) = \eta(r, \nu) - \chi(r, \nu) I(r, \mu, \nu)$$

- line absorption and emission coefficients

$$\chi(r, \nu) = \frac{\pi e^2}{m_e c} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right) \phi_{ij}(\nu) = \chi_L(r) \phi_{ij}(\nu)$$

$$\eta(r, \nu) = \frac{2h\nu^3}{c^2} \frac{\pi e^2}{m_e c} g_i f_{ij} \frac{n_j(r)}{g_j} \phi_{ij}(\nu) = \chi_L(r) S_L(r) \phi_{ij}(\nu)$$

$$\chi_L(r) = \frac{\pi e^2}{m_e c} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right)$$

CMF radiative transfer equation

Solution of the transfer equation for one line

$$-\frac{\nu v(r)}{c r} \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{dv(r)}{dr} \right) \frac{\partial}{\partial \nu} I(r, \mu, \nu) = \chi_L(r) \phi_{ij}(\nu) (S_L(r) - I(r, \mu, \nu))$$

- introduce a new variable

$$y = \int_{\nu}^{\infty} \phi_{ij}(\nu') d\nu'$$

- where
 - $y = 0$: the incoming side of the line
 - $y = 1$: the outgoing side of the line

CMF radiative transfer equation

Solution of the transfer equation for one line

$$-\frac{\nu v(r)}{c r} \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{dv(r)}{dr} \right) \frac{\partial}{\partial y} I(r, \mu, y) = \chi_L(r) \phi_{ij}(\nu) (S_L(r) - I(r, \mu, y))$$

- assumptions:
 - variables do not significantly vary with r within the “resonance zone” \Rightarrow
 - fixed r , $\frac{\partial}{\partial y} \rightarrow \frac{d}{dy}$
 - $\nu \rightarrow \nu_0$
- integration possible

CMF radiative transfer equation

Solution of the transfer equation for one line

$$I(y) = I_c(\mu)e^{-\tau(\mu)y} + S_L(1 - e^{-\tau(\mu)y})$$

- the Sobolev optical depth in spherical symmetry

$$\tau(\mu) = \frac{\chi_L(r)cr}{v_0v(r)\left(1 - \mu^2 + \frac{\mu^2r}{v(r)}\frac{dv(r)}{dr}\right)}$$

- the boundary condition is $I(y = 0) = I_c(\mu)$
- τ is given by the slope $\Rightarrow \tau \sim \left(\frac{dv}{dr}\right)^{-1}$

Radiative force

the radial component; force per unit of volume

$$f_{\text{rad}}(r) = \frac{1}{c} \int_0^{\infty} \chi(r, \nu) \mathcal{F}(r, \nu) d\nu$$

$$f_{\text{rad}}(r) = \frac{1}{c} \int_0^{\infty} \chi(r, \nu) d\nu \oint I(r, \nu, k) k d\Omega$$

$$f_{\text{rad}}(r) = \frac{2\pi}{c} \int_0^{\infty} \chi_L(r) \phi_{ij}(\nu) d\nu \int_{-1}^1 \mu I(r, \mu, \nu) d\mu$$

$$f_{\text{rad}}(r) = \frac{2\pi \chi_L(r)}{c} \int_0^1 dy \int_{-1}^1 \mu I(r, \mu, \nu) d\mu$$

Radiative force

the radial component; force per unit of volume

$$f_{\text{rad}}(r) = \frac{2\pi\chi_L(r)}{c} \int_0^1 dy \int_{-1}^1 \left[I_c(\mu) e^{-\tau(\mu)y} + S_L (1 - e^{-\tau(\mu)y}) \right] \mu d\mu$$

- where the Sobolev optical depth is

$$\tau(\mu) = \frac{\chi_L(r)cr}{v_0 v(r) \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{dv(r)}{dr} \right)}$$

- no net contribution of the emission to the radiative force (S_L is isotropic in the CMF)

$$f_{\text{rad}}(r) = \frac{2\pi\chi_L(r)}{c} \int_0^1 dy \int_{-1}^1 \mu I_c(\mu) e^{-\tau(\mu)y} d\mu$$

Radiative force

the radial component; force per unit of volume

$$f_{\text{rad}}(r) = \frac{2\pi \chi_L(r)}{c} \int_{-1}^1 \mu I_c(\mu) \frac{1 - e^{-\tau(\mu)y}}{\tau(\mu)} d\mu$$

- inserting

$$\tau(\mu) = \frac{\chi_L(r)cr}{v_0 v(r) \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{dv(r)}{dr}\right)}$$

$$f_{\text{rad}}(r) = \frac{2\pi v_0 v(r)}{r c^2} \int_{-1}^1 \mu I_c(\mu) \left[1 + \mu^2 \sigma(r)\right] \left\{1 - \exp\left[-\frac{\chi_L(r)cr}{v_0 v(r) (1 + \mu^2 \sigma(r))}\right]\right\} d\mu$$

$$\sigma(r) = \frac{r}{v(r)} \frac{dv(r)}{dr} - 1$$

Sobolev (1957), Castor (1974), Rybicki & Hummer (1978)

Radiative force

- Optically thin line

$$f_{\text{rad}}(r) = \frac{2\pi v_0 v(r)}{r c^2} \int_{-1}^1 \mu I_c(\mu) [1 + \mu^2 \sigma(r)] \left\{ 1 - \exp \left[-\frac{\chi_L(r) cr}{v_0 v(r) (1 + \mu^2 \sigma(r))} \right] \right\} d\mu$$

- Optically thin line

$$\frac{\chi_L(r) cr}{v_0 v(r) (1 + \mu^2 \sigma(r))} \ll 1$$

$$f_{\text{rad}}(r) \sim 1 - \exp \left[-\frac{\chi_L(r) cr}{v_0 v(r) (1 + \mu^2 \sigma(r))} \right] \approx \frac{\chi_L(r) cr}{v_0 v(r) (1 + \mu^2 \sigma(r))}$$

$$f_{\text{rad}}(r) = \frac{2\pi}{c} \int_{-1}^1 \mu I_c(\mu) \chi_L(r) d\mu$$

Radiative force

- Optically thin line

$$f_{\text{rad}}(r) = \frac{2\pi v_0 v(r)}{r c^2} \int_{-1}^1 \mu I_c(\mu) [1 + \mu^2 \sigma(r)] \left\{ 1 - \exp \left[-\frac{\chi_L(r) cr}{v_0 v(r) (1 + \mu^2 \sigma(r))} \right] \right\} d\mu$$

$$f_{\text{rad}}(r) = \frac{2\pi}{c} \int_{-1}^1 \mu I_c(\mu) \chi_L(r) d\mu$$

$$f_{\text{rad}}(r) = \frac{1}{c} \chi_L(r) \mathcal{F}(r)$$

- optically thin radiative force proportional to the radiative flux $\mathcal{F}(r)$
- optically thin radiative force proportional to the normalised line opacity $\chi_L(r)$ (or to the density)
- the same result as for the static medium

Radiative force

- Optically thick line

$$f_{\text{rad}}(r) = \frac{2\pi v_0 v(r)}{r c^2} \int_{-1}^1 \mu I_c(\mu) [1 + \mu^2 \sigma(r)] \left\{ 1 - \exp \left[-\frac{\chi_L(r) cr}{v_0 v(r) (1 + \mu^2 \sigma(r))} \right] \right\} d\mu$$

$$\frac{\chi_L(r) cr}{v_0 v(r) (1 + \mu^2 \sigma(r))} \gg 1$$

$$f_{\text{rad}}(r) \sim 1 - \exp \left[-\frac{\chi_L(r) cr}{v_0 v(r) (1 + \mu^2 \sigma(r))} \right] \approx 1$$

$$f_{\text{rad}}(r) = \frac{2\pi v_0 v(r)}{r c^2} \int_{-1}^1 \mu I_c(\mu) [1 + \mu^2 \sigma(r)] d\mu$$

Radiative force

- Optically thick line

$$f_{\text{rad}}(r) = \frac{2\pi v_0 v(r)}{r c^2} \int_{-1}^1 \mu I_c(\mu) [1 + \mu^2 \sigma(r)] d\mu$$

- neglect of the limb darkening:

- $\mu_* = \sqrt{1 - \frac{R_*}{r^2}}$

$$I_c(\mu) = \begin{cases} I_c = \text{const.} & \mu \geq \mu_* \\ 0, & \mu < \mu_* \end{cases}$$

- $\mathcal{F} = 2\pi \int_{\mu_*}^1 \mu I_c d\mu = \pi \frac{R_*}{r^2} I_c$

$$f_{\text{rad}}(r) = \frac{v_0 v(r) \mathcal{F}(r)}{r c^2} \left[1 + \sigma(r) \left(1 - \frac{1}{2} \frac{R_*}{r^2} \right) \right]$$

Radiative force

- Optically thick line

$$f_{\text{rad}}(r) = \frac{\nu_0 v(r) \mathcal{F}(r)}{r c^2} \left[1 + \sigma(r) \left(1 - \frac{1}{2} \frac{R_*}{r^2} \right) \right]$$

$$\sigma(r) = \frac{r}{v(r)} \frac{dv(r)}{dr} - 1$$

- large distance from the star: $r \gg R_*$

$$f_{\text{rad}}(r) = \frac{\nu_0 \mathcal{F}(r)}{c^2} \frac{dv(r)}{dr}$$

- optically thick radiative force proportional to the radiative flux $\mathcal{F}(r)$
- optically thick radiative force proportional to $dv(r)/dr$
- optically thick radiative force does not depend on the level populations (opacity) or the density