# EARTH'S ROTATION IRREGULARITIES DERIVED FROM $U T I_{B L I}$ BY METHOD OF MULTI-COMPOSING OF ORDINATES 

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#### Abstract

SUMMARY: Using the method of multi-composing of ordinates we have identified in Earth's rotation a long-periodic term with a period similar to the relaxation time of Chandler nutation. There was not enough information to assess its origin. We demonstrate that the method can be used even in the case when the data time span is comparable to the period of harmonic component.


Key words. Methods: data analysis - Methods: numerical - Time

## 1. LONG-PERIODIC TERMS

The ambiguity in explanation and representation of Chandler wobble is historical problem in Earth's rotation investigation. The wobble component of the spectrum of the changes in the Earth's rotation still presents a number of fascinating unresolved problems. On the other hand, there is general agreement on the annual and semiannual component (period), and their determinations are all consistent.

In the case of Chandler wobble (free nutation!) analyzes of the observational data can be divided into two groups: those assuming a one-parameter system, seeking a natural frequency for the nutation, and those characterizing the motion in a more intricate way.

In the list of problems the estimate of the Chandler wobble period is the first one. The second one is modelling and explaining of the existence of very complicated behavior of the amplitude of the wobble.

Analyzes by Pejović $(1984,1985)$ and Pejović and Šegan (1992) left us with dilemma: either the Chandler wobble is double peaked, or this is a
damped motion! Similar situation is with number of other papers (Kikuchi 1977, Lambeck 1980, Munk and MacDonald 1960). All of these analyzes situated the Chandler period in the interval between 1,13 and 1,21 years, and amplitude between the values of $0!12$ and $0!25$. Moreover, some of the authors used the same observational data, nevertheless obtaining the different results.

Melchior (1966) proposed time-variable model governed by three empirical laws. Jeffreys (1926) has concluded that free nutation is unstable: imperfect elasticity (visco-elasticity) assumes that under constant shear stress the strain increases with time. Stabilization is possible only under influence of some impulses (Vondrak and Pejović 1988, Pejović and Vondrak 1989), with corresponding intensity and regularity.

Classical Euler's equations lead to equations with damping factor (Chandler 1891a, 1891b, 1892):

$$
\begin{align*}
& \dot{x}+\mu x+\omega y=X(t) \\
& \dot{y}+\mu y-\omega x=Y(t) \tag{1}
\end{align*}
$$

where $\omega=2 \pi / T$ is circular frequency corresponding to the period $T, \mu$ is damping coefficient, $X(t)$
and $Y(t)$ are Earth's inertial factors. The solution is damped harmonic term with proper time given by $1 / \mu$ (time in which amplitude decreases $e$ times).

The dimensionless measure of damping commonly used is the dissipation factor $1 / Q$; if $\omega$ is frequency corresponding to tip of the peak, then $\mu$ describes peak's effective width. Then we have

$$
\begin{equation*}
Q=\frac{\omega}{2 \mu}=\frac{\pi f}{\mu} \tag{2}
\end{equation*}
$$

where $f$ is given in cycles/year.
The Chandler motion is not a stationary one. It may be described either by amplitude or by frequency modulations, but both of them have some shortcomings. We have accepted former one, assuming free nutation with unstable amplitude. Assume that Chandler motion is described by set of coordinates $(\xi, \eta)$; moreover, we will treat that motion as a two-dimensional stationary process:

$$
\begin{gather*}
d \xi=-\mu \xi d t-\omega \eta d t+d \nu  \tag{3}\\
d \eta=\omega \xi d t-\mu \eta d t+d \psi
\end{gather*}
$$

where $\mu$ and $\omega$ are the damping coefficient and the circular frequency, respectively, and $\nu(t)$ and $\psi(t)$ are independent stochastic processes (white noises) with

$$
\begin{gather*}
E(d \nu)=E(d \psi)=0 \\
E\left((d \nu)^{2}\right)=E\left((d \psi)^{2}\right)=a d t \tag{4}
\end{gather*}
$$

where $E$ is the statistical operator (mathematical expectation).

If

$$
\zeta=\xi+i \eta, \quad \chi=\nu+i \psi, \quad \gamma=\nu-i \omega
$$

system (3) is given by

$$
d \zeta=-\gamma \zeta d t+d \chi
$$

Broadening of spectral peaks always suggests the presence of damping mechanism with relaxation time

$$
\tau \approx(\pi \Delta \nu)^{-1}
$$

Many authors have estimated parameters of Chandler motion by maximum likelihood method. From the observational data covering 60 years, homogenized by Orlov's method (Куликов 1962), some authors deduced the following values:
$\omega=5,274 ;(T=1,191$ year $), \nu=0,06, \sigma_{T}=0,006$.
In this paper we attempt to isolate longperiodic term like relaxation time by a different method. The above value of $\nu=0,06$ indicates that relaxation time is about 16,5 years; results of other authors correspond to values of $Q$ in the range 30$60(\nu \approx(10,25)$ years $)$. These values are associated with possible tidal influence or solar activity influence (Đurović 1983, 1986). These values are anomalously low if the damping is to be attributed entirely to the inelasticity of the mantle.

## 2. THE DATA ANALYSIS

The data used in our analysis are collected by the Time service of Belgrade Astronomical Observatory in the time interval from 1964-1986. The data, homogenized by Jovanović (1993), are shown on Fig. 1.

From Fig. 1 it appears that there exist two groups of data: first group in the interval from March 1964 till July 1972, and the second one covering the interval from July 1972 to 1986. From both of them we have removed leap correction time (after 1972, this is a leap second).


Fig. 1. Homogenized data UT1-UTC collected by the Time service of Belgrade Astronomical Observatory between 1964. and 1986.

After unification and smoothing by means of Vondrak's method (Vondrak 1969) we have computed monthly mean values (see Figs. 2 and 3).


Fig. 2. Unifying data UT1-UTC (dotted line); Best fit smoothing curve (solid line).


Fig. 3. Monthly mean values of UT1-UTC.
These results are subsequently transformed by applying Labrouste's transformation $s_{8} s_{10} s_{12}$ by means of which all periodic terms under 2 years are damped (Šegan et al. 2003). The transformation has selectiveness presented on Fig. 4.


Fig. 4. Selectivity curve of transformation $s_{8} s_{10} s_{12}$.

The transformed values (see Fig. 5) suggest the existence of a long-periodic term with period of about 18 years, but there is some excitation also at position marked by an arrow. By plain transformation $s_{22}$ (see Fig. 6), we have removed this excited part.


Fig. 5. Monthly mean values of UT1-UTC after removing all periodic terms under 2 years.Marked position suggests excitation of some long-periodic term.


Fig. 6. Selectivity curve of transformation $s_{22}$.
After dividing resulting values with multiplication factor

$$
\begin{aligned}
& \sigma_{8} \sigma_{10} \sigma_{12} \sigma_{22}=\frac{\sin \left(\frac{17 \pi}{18 \times 365.25 / 30}\right) \times \sin \left(\frac{21 \pi}{18 \times 365.25 / 30}\right)}{\sin ^{4}\left(\frac{\pi}{18 \times 365.25 / 30}\right)} \times \\
& \quad \times \sin \left(\frac{25 \pi}{18 \times 365.25 / 30}\right) \times \sin \left(\frac{45 \pi}{18 \times 365.25 / 30}\right)
\end{aligned}
$$

we have obtained residual signal of (UT1-UTC) presented on Fig. 7.


Fig. 7. Residuals of UT1-UTC after removing existence of the long-periodic term.

Finally, we have calculated the period of this new harmonic term obtaining

$$
P=(17.74 \pm 0.09) \text { years }
$$

After removal of this term from the original (UT1-UTC) data, we arrived at the residuals presented on Fig. 8. FFT spectrum of these residuals is shown on Fig. 9. FFT spectrum of starting data is given on Fig. 10. The differences of the two are significant: the resonant effect of annual and Chandler term on Fig. 9 is smaller and, besides, we see a long-periodic term which cannot be identified by FFT applied to the starting data.


Fig. 8. Residuals of UT1-UTC after removing long-periodic term.


Fig. 9. FFT spectrum of data UT1-UTC after removing long-periodic term.


Fig. 10. FFT spectrum of starting data UT1-UTC.

## 3. CONCLUSIONS

The calculated value $P$ for the long-periodic term in (UT1-UTC) is similar as found in literature, but, being compatible with different possibilities, it does not provide enough information to explain its origin.

The reason may be in superposition of the long-periodic tidal component with resulting influence of the relaxation mechanisms.

The second important result is that the method of multi-composing of ordinates solves the boundary problem of FFT: the period of the considered term can be greater than the observational interval.

Finally, our results suggest that it is advisable to recompose a homogenized data from international ILS and IPMS services for new investigation.

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# НЕРАВНОМЕРНОСТИ ЗЕМЉИНЕ РОТАЦИЈЕ ИЗВЕДЕНЕ ИЗ $U T I_{B L I}$ МЕТОДАМА ВИШЕСТРУКИХ ТРАНСФОРМАЦИЈА ОР,ДИНАТА 

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Методама сложених трансформација ордината издвојен је дугопериодични члан Земљине ротације сличан релаксационом периоду Чендлерове нутације. Није било довољно информација да би се проценило његово

порекло. Ми приказујемо да метод може бити употребљив баш у случају када је распон временских података упоредив са периодом хармонијске компоненте.

