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NLTE MODEL ATMOSPHERES OF HOT STARS

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Abstract. Methods of the analysis of the atmospheres of hot stars are discussed with the emphasize to simultaneous solution of the equations of statistical equilibrium and radiative transfer both in static and moving media, including mass-loss rates determination.

1. INTRODUCTION

Stellar atmosphere is a layer, usually very thin compared to the radius of the star, at the outer boundary of the star. Since light is still the most important information source about stars and since all light from the star must come to us through the stellar atmosphere, the proper knowledge of processes there is of crucial importance.

In this paper we point out the attention to selected problems of the stellar atmosphere modelling both in static and moving atmospheres.

1. 1. STELLAR ATMOSPHERE PROBLEM

The problem of the construction of a model stellar atmosphere is usually considered as a task to find spatial distribution of various macroscopic quantities (e.g. temperature $T(\vec{r})$, electron density $n_e(\vec{r})$, mass density $\rho(\vec{r})$, etc.). It can also be viewed as a process of finding different microscopic distribution functions there. While for particle velocity distribution we usually assume equilibrium (Maxwellian) distribution, stellar radiation field has never the equilibrium (Planckian) distribution and we have to solve the radiative transfer equation to obtain the correct solution of the radiation field. Distribution of internal degrees of freedom (excitation and ionization stages) is usually assumed to be locally in equilibrium (the so-called LTE), however, in many situations this approximation is unusable and we have to solve the statistical equilibrium equations to determine the distribution of these internal degrees of freedom. The latter option is usually referred to as NLTE. A detailed discussion of LTE/NLTE options may be found in Hubený (1976) and in Mihalas (1978).

2. MODELLING OF STATIC ATMOSPHERES

Modelling of static stellar atmospheres has been described in detail, e.g., by Hubeny (1997). In modelling of static (generally NLTE) atmospheres we need to solve the equations of radiative transfer, radiative equilibrium, hydrostatic equilibrium, and statistical equilibrium simultaneously. Below we write these equations for the simplest case of a plane-parallel atmosphere, where the spatial dependence is restricted only to the z coordinate. The radiative transfer equation (which determines the specific intensity of radiation $I_{\mu\nu}$) for this case is

$$\mu \frac{\mathrm{d}I_{\mu\nu}(z)}{\mathrm{d}z} = \eta_\nu(z) - \chi_\nu(z) I_{\mu\nu}(z), \tag{1}$$

where η_ν is the emissivity, χ_ν is the opacity, and $\mu = \cos\theta$ is the cosine of the angle θ between the z-coordinate and the direction of propagation of radiation. The equation of radiative equilibrium (which determines the temperature structure T) reads

$$4\pi \int_0^\infty \left(\chi_\nu J_\nu - \eta_\nu\right) \mathrm{d}\nu = 0, \tag{2}$$

where J_ν is the mean radiation intensity. Introducing the column mass depth $\mathrm{d}m = -\rho\,\mathrm{d}z$ we can write the equation of hydrostatic equilibrium as

$$\frac{\mathrm{d}p}{\mathrm{d}m} = g - \frac{4\pi}{c} \int_0^\infty \frac{\chi_\nu}{\rho} H_\nu \,\mathrm{d}\nu \tag{3}$$

where p is the gas pressure, and H_ν is the Eddington flux. Finally, the equations of statistical equilibrium, which determine populations of individual atomic energy levels n_i ($i = 1, ..., NL$; NL is the number of explicit levels considered), can be written as (see Mihalas 1978)

$$n_i \sum_{\substack{l=1 \\ l\neq i}}^{NL} \left(R_{il} + C_{il}\right) + \sum_{\substack{l=1 \\ l\neq i}}^{NL} n_l \left(R_{li} + C_{li}\right) = 0, \tag{4}$$

where R_{il} and C_{il} are radiative and collisional rates, respectively. Detailed expressions for rates can be found in Mihalas (1978) and also in Kubát (2009). Opacity χ_ν and emissivity η_ν, which appear in equations (1) – (3), may be expressed as

$$\chi_\nu = \sum_i \sum_{j\neq i} \left[n_i - \frac{g_i}{g_j} n_j\right] \alpha_{ij}(\nu) + \sum_i \left(n_i - n_i^* e^{-\frac{h\nu}{kT}}\right) \alpha_{ik}(\nu) + \sum_k n_\mathrm{e} n_k \alpha_{kk}(\nu, T) \left(1 - e^{-\frac{h\nu}{kT}}\right) + n_\mathrm{e}\sigma_\mathrm{e} \tag{5}$$

$$\eta_\nu = \frac{2h\nu^3}{c^2} \left[\sum_i \sum_{j\neq i} n_j \frac{g_i}{g_j} \alpha_{ij}(\nu) + \sum_i n_i^* \alpha_{ik}(\nu) e^{-\frac{h\nu}{kT}} + \sum_k n_\mathrm{e} n_k \alpha_{kk}(\nu, T) e^{-\frac{h\nu}{kT}}\right], \tag{6}$$

where $\alpha_{ij}(\nu)$ is a cross-section of the transition $i \leftrightarrow j$, g_i is the statistical weight of the level i, n_e is the electron density, σ_e is the Thomson scattering cross section, and asterisks denote LTE values.

Since the system of equations (1) – (4) together with the expressions (5) and (6) is a system of nonlinear integro-differential equations, analytical solution of such system is impossible and we have to use some kind of numerical procedure.

We usually discretize the depth dependence of the atmosphere to *ND* depth points and the frequency spectrum is usually being represented by *NF* frequency points. If the statistical equilibrium equations are being solved for *NL* atomic energy levels and if we add temperature T and number particle density N, then the atmosphere at each depth point d may be represented by a vector $\vec{\psi}_d = (J_1, \ldots, J_{NF}, N, T, n_1, \ldots, n_{NL})$.

There are several methods how to solve such system of equations. Very efficient and robust method, a complete linearization method, was introduced by Auer and Mihalas (1969). This iterative method has been implemented for a solution of model atmosphere problem by a number of authors, e.g. Mihalas (1972) or Hubeny (1988). Applying the accelerated lambda iteration method (ALI) we may express the radiation field using quantities from previous iterations,

$$J_\nu^{(n)} = \Lambda^* S_\nu^{(n)} + (\Lambda - \Lambda^*)\, S_\nu^{(n-1)}. \tag{7}$$

Thus we are able to remove the radiation field from the linearization step and significantly reduce the size of inverted matrices. More details about implementation of ALI and further references may be found in Kubát (2003).

3. MODELLING OF EXPANDING ATMOSPHERES

Compared to the static case, the situation is more complicated for the case of expanding atmospheres. There a simultaneous solution of both the stellar atmosphere and wind is necessary. The radiative transfer equation has to be solved in a moving medium, where the absorption coefficient depends on angle (in an observer frame) thanks to the Doppler effect. In addition, instead of the hydrostatic equilibrium equation we have to solve the equations of motion and continuity, which then determine the density and velocity structure of the atmosphere.

The atmosphere is significantly supported by the radiation pressure. For the case of hot stars, an important part of this pressure (or force) is caused by absorption and scattering in a number of spectral lines, mostly in the UV spectral region. A substantial complication arises from properties of radiative force, since it depends nonlinearly on the velocity gradient.

Due to the complexity of the problem being solved, a number of approximation is usually being used. First, it is often assumed that the atmosphere consists of two distinct parts, namely the static atmosphere and the expanding wind, which is located above the atmosphere. This approximation is usually referred to as the core-halo approximation. Additional common approximation is applied when we aim at solving the so-called NLTE problem in the wind (in the "halo" part). In these calculations, the velocity and density structure is held fixed. In addition, treatment of lines is many times being simplified by using the Sobolev approximation, while for the continuum radiative transfer the same method as for the static case may be used.

The above mentioned dependence of the radiative force on a huge number of UV spectral lines makes their direct treatment computationally expensive. That's why the radiative force is usually approximated using the so-called force multipliers, which enable fast calculation of the radiative force without the necessity to solve the radiative transfer equation.

3. 1. HYDRODYNAMIC CALCULATIONS AND DRIVING FORCE

The importance of the influence of the radiative force on individual atoms was first discussed by Johnson (1925) and Milne (1926). The concept of a stellar wind driven by line absorption was developed by Lucy and Solomon (1970). Castor (1974) did detailed calculations of the line force produced by spectral lines and subsequently, Castor, Abbott and Klein (1975, hereafter CAK) performed first hydrodynamical solution of the line driven wind. They used a parameterized form of the radiative force

$$f_{\mathrm{rad}} = \frac{\sigma_{\mathrm{e}} F}{c} k t^{-\alpha} \left(\frac{n_e}{W}\right)^{\delta} \tag{8}$$

using parameters k, α (usually called force multipliers) while the multiplier δ was introduced later by Abbott (1982). CAK solution of the line driven wind initiated intensive research in this field. Pauldrach (1987) did huge NLTE calculations of the radiative force, which were later used in analysis of hot star winds (Pauldrach et al. 1990, 1994). Vink et al. (1999) determined the radiative force using NLTE Monte Carlo calculations. Hydrodynamic calculations without force multipliers in Sobolev approximation were done by Krtička and Kubát (2004).

As mentioned above, first wind solution was calculated by CAK. Later Abbott (1980) analyzed the solution of the hydrodynamic calculation in a more detail and discovered the radiative-acoustic waves using stability analysis. In the above mentioned first hydrodynamical calculations the star was approximated as a point source. A more realistic approach, which takes into account full stellar surface, was applied by Pauldrach, Puls and Kudritzki (1986) and Friend and Abbott (1986), while the latter authors took also rotation into account. A stride forward was made by Owocki, Castor and Rybicki (1988) and later by Feldmeier (1995), who solved the time evolution of the wind using hydrodynamical simulations.

Radiation acts directly only on metals, while the radiative force acting on hydrogen and helium is negligible. The latter atoms are accelerated by collisions with metals. This multicomponent nature of the wind was first taken into account by Springmann and Pauldrach (1992), stationary models were calculated by Krtička and Kubát (2000). Hydrodynamical simulations of a multicomponent wind were performed by Votruba et al. (2007). Multicomponent nature of the line driven winds was reviewed by Krtička and Kubát (2007).

3. 2. EXTENDED HYDRODYNAMIC CALCULATIONS

More advanced calculations combine hydrodynamical calculations with solution of other equations. Standard hydrodynamical calculations solve equations of continuity and motion, which give us density and velocity structure, respectively. Sometimes they are extended by an energy equation, which determines the temperature structure. These equations are iteratively combined with the solution of the radiative transfer equation (in Sobolev approximation, in comoving frame, or using Monte Carlo approach), often supplemented by statistical equilibrium equations, which means solution of the so-called NLTE problem. The radiative transfer calculations enable subsequent recalculation of the radiative force and such models are iterated until satisfactory convergence is reached. As examples may serve calculations by Gabler et al. (1989), Schaerer and Schmutz (1994), Pauldrach et al. (1994, 2001), Krtička and Kubát (2004), and Gräfener and Hamann (2005).

4. MASS-LOSS RATE DETERMINATION

Compared to the static case, where basic stellar parameters are the stellar mass M_*, stellar radius R_*, and luminosity L, additional basic parameters of a moving atmosphere are the terminal wind velocity v_∞ and the mass-loss rate $\dot{M}$. While the measurement of the former is relatively straightforward for the case of P-Cygni line profiles, the mass-loss rate determination is still subject to many uncertainties.

4. 1. β-VELOCITY LAW

A commonly used simplification of the velocity structure of the atmosphere, which enables to avoid solution of the momentum equation, is the so-called β-velocity law. It is a simple power-law dependence of the radial expansion velocity on radius in the form

$$v = v_\infty \left(1 - \frac{R_*}{r}\right)^\beta . \tag{9}$$

It was introduced already by Milne (1926) and Chandrasekhar (1934) with $\beta = 1/2$. Typical value for line driven stellar winds is $\beta \approx 0.8$, but values of $\beta \sim 3$ were also found for some stars (e.g. Evans et al. 2004). The value of v_∞ can be determined from observed P Cygni line profiles.

4. 2. RADIO MASS-LOSS RATE DETERMINATION

Radio emission is mainly of thermal origin caused by free-free radiation produced in the outer parts of the wind. Radio measurements are used as diagnostic tools to obtain mass-loss rate. In this method, following assumptions are usually applied: gas is completely ionized everywhere, electron temperature is constant, spherical symmetry and LTE are assumed, expanding velocity is constant and equal to the terminal velocity v_∞, and the radial n_e-distribution is power-law. Measured radio flux (F_ν) is related to the mass-loss rate ($\dot{M}$) as in the following expression (Wright and Barlow 1975)

$$F_\nu = 23.2 \left(\frac{Z\dot{M}}{\mu v_\infty}\right)^{4/3} \frac{\left[\nu\gamma g_{\rm ff}\left(\nu, T\right)\right]^{2/3}}{D^2} \left[Jy\right], \tag{10}$$

where μ is the mean ionic weight (in a.m.u.), v_∞ is the terminal wind velocity in $\mathrm{km\,s^{-1}}$, D is the distance to the star in kpc, ν is the frequency in Hz, $\gamma = n_e/n_i$ is the mean number of electrons per ion, Z is the mean charge per ion, and $g_{\rm ff}$ is the free-free Gaunt factor.

Detailed derivation of expression (10) can be found in Wright and Barlow (1975) and Panagia and Felli (1975). From expression (10), stellar mass-lose rate can be expressed as

$$\dot{M} = 0.095 \frac{\mu v_\infty F_\nu^{3/4} D^{3/2}}{Z\left(\gamma g_{\rm ff}\nu\right)^{1/2}} \left[\frac{M_\odot}{\rm yr}\right]. \tag{11}$$

In the winds of OB stars we may use $\mu = 1.25$, $Z = 1.0$, $\gamma = 1.0$, $n_e \approx n_i$ and $\nu = 4.88$ GHz, then mass-loss rate is given by (Abbott et al. 1981)

$$\dot{M} = 1.7 \cdot 10^{-6} \frac{v_\infty F_\nu^{3/4} D^{3/2}}{g_{\rm ff}^{1/2}} \left[\frac{M_\odot}{\rm yr}\right], \tag{12}$$

where v_∞, F_ν, and D can be derived from observation and mass-loss rate can be easily calculated.

Radio measurement is known as one of the most reliable method for mass-loss rate determination and this method is free of uncertain assumptions. But on the other hand, there is problem for stars which are too distant. In this case, flux densities are low and, consequently, only about twenty stars have accurate flux density determinations (Scuderi et al. 1998). Another problem is that part of radio emission may be of non-thermal origin (Bieging et al. 1989). Because of this, other spectral regions also have to be used for more accurate mass-loss rate determination.

4. 3. Hα MASS-LOSS RATE DETERMINATION

Hα emission originates in regions closer to the star than the free-free emission. In this method of mass-loss rate determination usually a β-velocity law is assumed with some specific value of β, and then the theoretical emergent Hα line profile is calculated. The mass-loss rate corresponding to the model with the best fit is then called the *observed mass loss rate.*

First NLTE line profiles from radiatively driven winds (for a CAK velocity structure) were calculated by Klein and Castor (1978). However, for mass-loss rate determination they used only comparison of observed and theoretical equivalent widths. Hα line profiles for several different prescribed velocity fields were calculated by Olson and Ebbets (1981). Leitherer (1988) determined mass-loss rates for a number of stars using Hα equivalent widths, while Puls et al. (1996) used detailed Hα profiles for mass-loss rate determination.

4. 4. MASS-LOSS RATES FROM UV RESONANCE LINES

Besides Hα and radio measurements of mass-loss rates also ultraviolet resonance lines are indicators of mass-loss from early type stars. However, only unsaturated lines can be used for this purpose. In addition, knowledge of the ionization structure of the wind is necessary (cf. Lamers et al. 1999). As an example, Fullerton, Massa and Prinja (2006) used the resonance lines of P V $\lambda\lambda$ 1118, 1128 Å as extremely suitable for this purpose. However, they found discrepancies between the three above mentioned mass-loss rates determinations.

4. 5. MASS-LOSS RATES FROM MODEL ATMOSPHERES

The most sophisticated method of mass-loss rate measurements is that using detailed model atmosphere analysis. There are several codes available, with different sophistication of the radiative transfer solution. The codes CMFGEN (Hillier and Miller 1998) and PoWR (Gräfener and Hamann 2005) use solution of the radiative transfer equation in a comoving frame. The latter code takes into account the consistent hydrodynamical structure by means of iterative recalculation (see Section 3.2). The code WM-basic (Pauldrach et al. 2001) also recalculates the hydrodynamical structure iteratively, but uses the Sobolev approximation for the solution of the transfer equation in lines. Line blanketing and line blocking are correctly taken into account by CMFGEN, but assuming the prescribed velocity profile, which is the β-law in many cases. The code FASTWIND (Santolaya-Rey et al. 1997, Puls et al. 2005) uses a number of approximations that make the calculations of emergent radiation significantly faster while preserving the most important properties of the atmosphere. A useful comparison of CMFGEN, WM-basic, and FASTWIND codes was presented by Puls et al. (2005).

The above mentioned codes were applied to a number of stars to determine their mass loss rates, e.g. Bouret et al. (2003), Evans et al. (2004), Trundle et al. (2004), Crowther et al. (2006), and many other.

4. 6. CLUMPING IN HOT STAR WINDS

There is a vast observational evidence of absorption or emission features moving across line profiles. Such moving bumps may be explained as corotating interation regions (Mullan 1984, Cranmer and Owocki 1986) or as a consequence of clumping. From other side, mass loss rates determined from Hα and radio measurements sometimes disagree. These facts lead to a hypothesis that hot star winds are clumped (Runacres and Blomme 1996, Blomme and Runacres 1997).

Clumps are regions with larger density than the surrounding wind. Consequently, clumped medium has lower opacity and this causes that the radiative force changes. Solution of the radiative transfer equation in such medium is not an easy task and and efficient method, which is able to cope with such inhomogeneities, must be used. Clumps are usually assumed to form as a consequence of shocks arising from wind instabilities (see Feldmeier et al. 2008, and references therein).

5. SUMMARY

While for static atmospheres, both for the plane-parallel and spherically symmetric cases, modelling is used almost routinely, for the case of moving atmospheres the latter is more complicated. This task is usually split to two parts, namely to hydrodynamic calculations with simplified treatment of radiation field and to NLTE modelling for known hydrodynamic structure, often simplified using the β-velocity law. NLTE modelling is often being used for mass-loss rate determination.

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References

Abbott, D. C.: 1980, *Astrophys. J.*, **242**, 1183.
Abbott, D. C.: 1982, *Astrophys. J.*, **259**, 282.
Abbott, D. C., Bieging, J. H. and Churchwell, E.: 1981, *Astrophys. J.*, **250**, 645.
Auer, L. H. and Mihalas, D. : 1969, *Astrophys. J.*, **158**, 641.
Bieging, J. H., Abbott, D. C., Churchwell, E. B.: 1989, *Astrophys. J.*, **340**, 518.
Blomme, R. and Runacres, M. C.: 1997, *Astron. Astrophys.*, **323**, 886.
Bouret, J.-C., Lanz, T., Hillier, D. J. et al.: 2003, *Astrophys. J.* **592**, 1182.
Castor, J. I.: 1974, *Mon. Not. R. Astron. Soc.*, **169**, 279.
Castor, J. I., Abbott, D. C. and Klein, R. I.: 1975, *Astrophys. J.*, **195**, 157.
Chandrasekhar, S.: 1934, *Mon. Not. R. Astron. Soc.*, **94**, 522.
Cranmer, S. R. and Owocki, S. P.: 1986, *Astrophys. J.*, **462**, 469.
Crowther, P. A., Lennon, D. J. and Walborn, N. R.: 2006, *Astron. Astrophys.*, **446**, 279.

Evans, C. J., Crowther, P. A., Fullerton, A. W. and Hillier, D. J.: 2004, *Astrophys. J.*, **610**, 1021.
Feldmeier, A.: 1995, *Astron. Astrophys.*, **299**, 523.
Feldmeier, A., Hamann, W.-R., Rätzel, D. and Oskinova, L. M.: 2008, in Clumping in Hot Star Winds, W.-R. Hamann, A. Feldmeier and L. M. Oskinova eds., Universität Verlag, Potsdam, p. 115.
Friend, D. B. and Abbott, D. C.: 1986, *Astrophys. J.*, **311**, 701.
Fullerton, A. W., Massa, D. L. and Prinja, R. K.: 2006, *Astrophys. J.*, **637**, 1025.
Gabler, R., Gabler, A., Kudritzki, R.-P., Puls, J. and Pauldrach, A.: 1989, *Astron. Astrophys.*, **226**, 162.
Gräfener, G. and Hamann, W.-R.: 2005, *Astron. Astrophys.*, **432**, 633.
Hillier, D. J. and Miller, D. L.: 1998, *Astrophys. J.*, **496**, 407.
Hubený, I.: 1976, kandidátská disertační práce, AsÚ ČSAV Ondřejov.
Hubeny, I.: 1988, *Comput. Phys. Commun.*, **52**, 103
Hubeny, I.: 1997, in Stellar Atmospheres: Theory and Observations, J. P. De Greve, R. Blomme and H. Hensberge eds., Lecture Notes in Physics Vol. 497, Springer Verlag, Berlin, p. 1.
Johnson, M. C.: 1925, *Mon. Not. R. Astron. Soc.*, **85**, 813.
Klein, R. I. and Castor, J. I.: 1978, *Astrophys. J.*, **220**, 902.
Krtička, J. and Kubát, J.: 2000, *Astron. Astrophys.*, **359**, 983.
Krtička, J. and Kubát, J.: 2004, *Astron. Astrophys.*, **417**, 1003.
Krtička, J. and Kubát, J.: 2007, in Active OB-Stars: Laboratories for Stellar & Circumstellar Physics, S. Štefl, S. P. Owocki and A. T. Okazaki eds., ASP Conf. Ser. Vol. 361, p. 153.
Kubát, J.: 2003, in Modelling of Stellar Atmospheres, IAU Symp. 210, N. E. Piskunov, W. W. Weiss and D. F. Gray eds., Astron. Soc. Pacific, A8.
Kubát, J.: 2009, in Non-LTE Line Formation for Trace Elements in Stellar Atmospheres, R. Monier et al. eds., EAS Publ. Ser., in press.
Lamers, H. J. G. L. M., Haser, S., de Koter, A. and Leitherer, C.: 1999, *Astrophys. J.*, **516**, 872
Leitherer, C.: 1988, *Astrophys. J.*, **326**, 356.
Lucy, L. B. and Solomon, P. M.: 1970, *Astrophys. J.*, **159**, 879.
Mihalas, D.: 1972, NCAR-TN/STR-76, NCAR Boulder.
Mihalas, D.: 1978, Stellar Atmospheres, 2nd ed., W. H. Freeman & Comp., San Francisco.
Milne, E. A.: 1926, *Mon. Not. R. Astron. Soc.*, **86**, 459.
Mullan, D. J.: 1984, *Astrophys. J.*, **283**, 303.
Olson, G. L. and Ebbets, D.: 1981, *Astrophys. J.*, **248**, 1021.
Owocki, S. P., Castor, J. I. and Rybicki, G. B.: 1988, *Astron. Astrophys.*, **335**, 914.
Panagia, N. and Felli, M.: 1975, *Astron. Astrophys.*, **39**, 1.
Pauldrach, A.: 1987, *Astron. Astrophys.*, **183**, 295.
Pauldrach, A., Puls, J. and Kudritzki, R. P.: 1986, *Astron. Astrophys.*, **164**, 86.
Pauldrach, A. W. A., Kudritzki, R. P., Puls, J. and Butler, K.: 1990, *Astron. Astrophys.*, **228**, 125.
Pauldrach, A. W. A., Kudritzki, R. P., Puls, J., Butler, K. and Hunsinger, J.: 1994, *Astron. Astrophys.*, **283**, 525.
Pauldrach, A. W. A., Hoffmann, T. L. and Lennon M.: 2001, *Astron. Astrophys.*, **375**, 161.
Puls, J., Kudritzki, R.-P., Herrero, A. et al.: 1996, *Astron. Astrophys.*, **305**, 171.
Puls, J., Urbaneja, M. A., Venero, R. et al.: 2005, *Astron. Astrophys.*, **435**, 669.
Runacres, M. C. and Blomme, R.: 1996, *Astron. Astrophys.*, **309**, 544.
Santolaya-Rey, A. E., Puls, J. and Herrero, A.: 1997, *Astron. Astrophys.*, **323**, 488.
Schaerer, D. and Schmutz, W.: 1994, *Astron. Astrophys.*, **288**, 231.
Scuderi, S., Panagia, N., Stanghellini, C., Trigilio, C. and Umana, G.: 1998, *Astron. Astrophys.*, **332**, 251.
Springmann, U. W. E. and Pauldrach, A. W. A.: 1992, *Astron. Astrophys.*, **262**, 515.
Trundle, C., Lennon, D.J., Puls J. and Dufton, P. L.: 2004, *Astron. Astrophys.*, **417**, 217.
Vink, J. S., de Koter, A. and Lamers, H. J. G. L. M.: 1999, *Astron. Astrophys.*, **350**, 181.
Votruba, V., Feldmeier, A., Kubát, J., Rätzel, D.: 2007, *Astron. Astrophys.*, **474**, 549.
Wright, A. E., Barlow, M. J.: 1975, *Mon. Not. R. Astron. Soc.*, **170**, 41.